THE JOURNAL of the Acoustical Society of America

• CONTENTS •

Transmission, Reflection, and Guiding of an Exponential Pulse by a Steel Plate in Water. I. Theory . . . . . M. F. M. Osborne and S. D. Hart 1

Wave-Front Determination in a Unidirectional Supersonic Beam . . . . . L. W. Labaw 19

Observed Classical Sound Absorption in Air . . . . . W. H. Pielemeier 24

Working Standard for Sound Pressure Measurements . . . . . Frank Massa 29

Application of the Wave Theory of Room Acoustics to the Measurement of Acoustic Impedance . . . . . Cyril M. Harris 35

Frequency Standard for Use at High and Low Frequencies . . . . . Rogelio P. McLoughlin, translated by Neil E. Handel 46

Investigation of Resonances in Filaments by Mechanical Actuation and Measurement of Electromagnetically Induced Voltages . . . . . R. W. Carlisle and H. W. Koren 71

Group Audiometry . . . . . J. Donald Harris 73

Clinical Phenomena in Conductive Media: The Individual Earpiece . . Mayer B. A. Schier 77

Musicology, the Stepchild of the Sciences . . . . . Abe Pepinsky 83

Letter to the Editor . . . . . Charles Williamson 86

Acoustical Society News 87

Current Publications on Acoustics . . . . . F. A. Firestone 89

- Book Review 89
- References to Contemporary Papers on Acoustics 90
- Review of Acoustical Patents . . . . . Robert W. Young 92

Program of the Thirtieth Meeting of the Acoustical Society of America, Hotel Pennsylvania, New York, New York, May 11-12, 1945 100

Vol. 17, No. 1 JULY, 1945

THE JOURNAL
OF THE
ACOUSTICAL SOCIETY OF AMERICA

Volume 17 Number 1

JULY • 1945

Transmission, Reflection, and Guiding of an Exponential Pulse by a Steel Plate in Water. I. Theory

M. F. M. Osborne and S. D. Hart,
Naval Research Laboratory, Washington, D. C.

(Received April 14, 1945)

In this paper the problem of the interaction of an underwater explosion wave with a steel plate is discussed. Particular attention is given to those aspects of the problem in which the plate can be considered as an elastic, two-dimensional wave guide. The phase velocities of the more important modes of the plate are evaluated as functions of frequency. They are used to derive the properties of the precursor, an oscillation which precedes the explosion wave as it travels along the plate. The results of the theory are compared briefly with experiment. A more detailed discussion of the experiments will be given in a second report. The methods used in this report are also applicable to the evaluation of the phase velocities of the modes of an electromagnetic-wave guide. The propagation of a transient, such as an explosion wave, down a wave guide, presents the interesting mathematical problem of the evaluation of a contour integral of a function of a complex variable defined implicitly. No rigorous solution for this problem as yet exists.

I. INTRODUCTION

IN a second paper (II) under the above title, an account will be given of experiments measuring the pressure in the neighborhood of a steel disk in water, caused by a No. 8 Dupont blasting cap. In this paper is given a theoretical study of the same problem. This study is primarily intended to give a theoretical explanation of the "precursor." This is a characteristic oscillation which, for oblique angles of incidence, preceded the explosion wave as it traversed the diameter of the disk, as is shown in Fig. 14.

All the experimental phenomena observed can be explained on the basis of theory already developed, except the existence and properties of the precursor. In this report the problem of the action of an exponential transient (the explosion wave) on a steel plate will be discussed, with special attention to those aspects of the problem concerned with the precursor. Essentially, the problem to be solved is that of the two-dimensional wave guide formed by an elastic plate in water.

II. STATEMENT OF THE PROBLEM

The problem will first be stated in complete generality, and then simplified and approximated

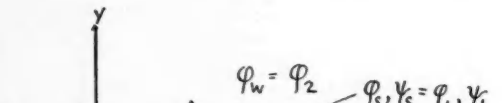


FIG. 1. Steel disk approximated by a semi-infinite slab.

until it becomes tractable. It must be remembered that a complete solution is not desired, but only those properties of the solution which can be compared with experiments. In particular it is desired to know the pressure, as a function of time for a specified position, near the disk, before the arrival of the main shock, i.e., the properties of the precursor. By "properties" is meant the approximate period of the pressure fluctuations, when they appear, and how long they last.

Let the displacement of the medium be given by

$$\begin{aligned} \mathbf{r}_s &= \text{grad } \varphi_s + \text{curl } \psi_s, \\ \mathbf{r}_w &= \text{grad } \varphi_w, \end{aligned} \tag{1}$$

where the subscripts w, s, refer to the water and steel. It is required to find functions φ and ψ satisfying the wave equations

$$\begin{aligned} \left(\nabla^2 - \frac{1}{c_r^2}\frac{\partial^2}{\partial t^2}\right)\varphi_s &= 0, \\ \left(\nabla^2 - \frac{1}{c_d^2}\frac{\partial^2}{\partial t^2}\right)\psi_s &= 0, \\ \left(\nabla^2 - \frac{1}{c_w^2}\frac{\partial^2}{\partial t^2}\right)\varphi_w &= 0, \end{aligned} \tag{2}$$

subject to the boundary conditions: r_n continuous on the water-steel boundary, p_n continuous on the water-steel boundary, $p_t = 0$ in water and on the water-steel boundary. r_n is the normal displacement in the medium, and p_n and p_t the normal and tangential stresses. The notation follows Love[1] and Lamb.[2]

[1] A. E. H. Love, *The Mathematical Theory of Elasticity* (Cambridge University Press, New York, 1927), fourth edition, p. 309.

[2] H. Lamb, Proc. Roy. Soc. London **A93**, 114 (1917).

The initial conditions are set by the requirement that the incident wave be a diverging wave of exponential form. If time is measured from the instant that the explosion wave just touches the plate, then at $t=0$,

$$\begin{aligned} p_n &= [\exp(r/c_w\tau)]/r, \quad r<R; \\ &= 0, \quad r>R, \end{aligned} \tag{3}$$

where r is the distance coordinate measured from the explosion source, and R the distance to the nearest point of the plate, τ is the "time constant" of the exponential explosion wave, and c_w the velocity of sound in water.

This problem is first simplified by replacing it by a two-dimensional one, since in the experiments the distance to the explosion source, 11 ft. 6 in., was large compared to the radius of the disk, 1 ft. The disk is taken as a semi-infinite slab, the incident wave is now plane instead of spherical, and the motion takes place in two dimensions only (Fig. 1). This approximation can be expected to be best up to a time shortly after the arrival of the explosion wave in the water, and to describe disturbances whose dimensions are small compared to the radius of the disk.

The problem can be still further simplified. Since the observations refer to conditions close to the plate, i.e., at points such as A, A', one would like to determine φ_w at these points as functions of time, and at various distances from the edge BB'. It is known from the observations that the diffracted wave is small. The problem of determining φ_w at A, or A', is therefore divided into two parts: (1) A part caused by the transmission of the wave through the plate, or its reflection from the plate, considered as infinite and (2) a part arising from the forces or displacements obtaining initially at the edge of the plate, BB', caused by the explosion wave.

The first problem is then one of the reflection or transmission of a plane-wave transient at an infinite plane plate. The second problem is that of the plate considered as a two-dimensional "wave guide," with specified initial conditions. Both of these problems can be solved by first obtaining the solution for steady-state sinusoidal time dependence, and then superposing by integration solutions for different frequencies, in order to represent the incident transient, or initial conditions.

The first problem, the reflection and transmission of sinusoidal waves at an infinite plate, has been solved by Reissner.[3] He assumes an incident wave of amplitude unity, and of arbitrary frequency and angle of incidence θ. In order to satisfy the six boundary conditions, three on each side of the plate, and the conditions that the amplitudes of all the progressive waves set up be independent of the coordinate along the plate (x) and time, just six additional waves with amplitudes different from zero must be found. There is one reflected wave, four waves inside the plate (one dilation, one rotation, in each direction $\pm y$), and one transmitted wave. There are therefore seven wave amplitudes (one specified as unity), and six boundary conditions. These boundary conditions give a set of six inhomogeneous equations on the six unknown amplitudes, which are thus completely determined. In analogy with the corresponding electromagnetic problem of reflection from a lamina, this may be called the inhomogeneous problem of waves in an elastic plate in water.

The homogeneous problem is the one in which the plate is considered a two-dimensional wave guide. Here there are four waves inside the plate, and one only in the water on each side of the plate. With six waves in all, the six boundary conditions give a set of six homogeneous linear equations to be satisfied by the wave amplitudes. There is therefore no solution unless the determinant of the coefficients of the amplitudes vanishes. This determinant determines the normal modes, or a discrete set of space distribution constants, for any specified frequency. Only one of the six amplitudes can be specified arbitrarily; the rest are determined.

III. THE INHOMOGENEOUS CASE

Reissner's solution to the inhomogeneous problem can be used to determine the shape of the transient reflected or transmitted through the plate as follows. He obtains the amplitudes $A_{t\theta}(\omega)$ and $A_{r\theta}(\omega)$ of the transmitted and reflected waves when a sinusoid of amplitude unity, circular frequency ω, at angle θ, is incident on the plate. If a plane-wave transient at an angle of incidence θ has the form or profile (ignoring the dependence on position)

$$f_\theta(t)=e^{-t/\tau}, \quad t>0;$$
$$=0, \quad t<0, \tag{4}$$

then the reflected wave is

$$R(t)=\frac{1}{(2\pi)^{\frac{1}{2}}}\int_{-\infty}^{\infty} g_\theta(\omega)A_{r\theta}(\omega)e^{-i\omega t}d\omega, \tag{5}$$

and the transmitted wave is

$$T(t)=\frac{1}{(2\pi)^{\frac{1}{2}}}\int_{-\infty}^{\infty} g_\theta(\omega)A_{t\theta}(\omega)e^{-i\omega t}d\omega, \tag{6}$$

where $g_\theta(\omega)$ is the Fourier transform of $f_\theta(t)$

$$g_\theta(\omega)=\frac{1}{(2\pi)^{\frac{1}{2}}}\int_{-\infty}^{\infty} f_\theta(t)e^{i\omega t}d\omega. \tag{7}$$

The effect of the functions $A_{t\theta}(\omega)$, $A_{r\theta}(\omega)$ in cutting out parts of the Fourier spectrum is as follows. If $A_{t\theta}(\omega)$ is small (compared to unity) at high frequencies, and of order unity at low frequencies, the shock front of the transmitted exponential wave will be rounded off and the height of the peak diminished. If $A_{r\theta}(\omega)$ is of order unity at high frequencies and small at low frequencies, the reflected wave preserves its steep shock front, but falls off more rapidly than the incident wave and may even become negative. These effects are illustrated qualitatively in Fig. 2, and in Fig. 14, Nos. 2, 1. Detailed data and discussion will be given in the second report (II) on this work. The oscillograms of Fig. 14 were obtained with a tourmaline piezoelectric pressure gauge, broadband television amplifier, and cathode-ray oscillograph.

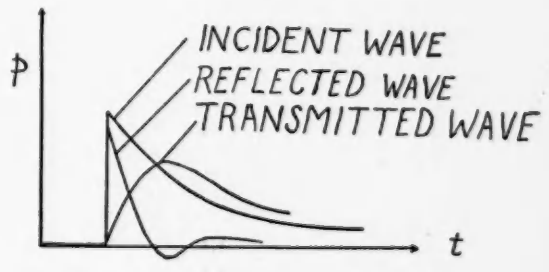


FIG. 2. Schematic incident, reflected, and transmitted wave.

[3] H. Reissner, Helv. Phys. Acta **2**, 140 (1938).

IV. THE HOMOGENEOUS CASE

The homogeneous, or wave guide problem will form the main subject of this report, and will be used to derive some properties of the precursor. It will therefore be stated more completely than the inhomogeneous case discussed above. The notation and method will follow that of Lamb,[2] where the problem of a plate in a vacuum is solved. Some familiarity with Lamb's paper must be assumed, as his solution was used as a basis for solving the present problem. The two are closely related.

The steady state wave guide problem may be stated as follows. Find the functions φ and ψ such that (Fig. 1):

In the water,

$$\nabla^2\varphi_0-\frac{1}{c_w^2}\frac{\partial^2\varphi_0}{\partial t^2}=0,$$
$$\nabla^2\varphi_2-\frac{1}{c_w^2}\frac{\partial^2\varphi_2}{\partial t^2}=0. \tag{8}$$

In the plate,

$$\nabla^2\varphi_1-\frac{1}{c_d^2}\frac{\partial^2\varphi_1}{\partial t^2}=0,$$
$$\nabla^2\psi_1-\frac{1}{c_r^2}\frac{\partial^2\psi_1}{\partial t^2}=0. \tag{9}$$

Displacement in x,

$$u=\frac{\partial\varphi_1}{\partial x}+\frac{\partial\psi_1}{\partial y}$$

Displacement in y,

$$v=\frac{\partial\varphi_1}{\partial y}-\frac{\partial\psi_1}{\partial x}$$

} in the plate. (10)

In the water,

$$u=\frac{\partial\varphi_{0\text{ or }2}}{\partial x},\quad v=\frac{\partial\varphi_{0\text{ or }2}}{\partial y}.$$

Velocity of dilational waves in the plate,

$$c_d=((\lambda_s+2\mu_s)/\rho_s)^{\frac{1}{2}}=0.57\cdot10^6\text{ cm/sec}.$$

Velocity of rotational waves in the plate,

$$c_r=(\mu_s/\rho_s)^{\frac{1}{2}}=0.33\cdot10^6\text{ cm/sec}. \tag{11}$$

Velocity of dilational (sound) waves in the water,

$$c_w=(\lambda_w/\rho_w)^{\frac{1}{2}}=0.15\cdot10^6\text{ cm/sec}.$$

λ and μ are Lamé's constants, and Poisson's assumption that $\lambda_s=\mu_s$ is made for the plate. $\rho_w=1.0$, $\rho_s=7.6$ g/cm^3. The half-thickness, f, of the plate is 0.138 cm.

Boundary conditions at $y=\pm f$.

(1) p_{yy} is continuous at $y=\pm f$.

$$(p_{yy})_1=(p_{yy})_0 \quad\text{at}\quad y=-f.$$
$$(p_{yy})_1=(p_{yy})_2 \quad\text{at}\quad y=+f. \tag{12}$$
$$p_{yy}=\lambda(\partial u/\partial x+\partial v/\partial y)+2\mu\partial v/\partial y.$$

(2) p_{xy} vanishes at $y=\pm f$.

$$(p_{xy})_1=0 \quad\text{at}\quad y=\pm f.$$
$$p_{xy}=\mu(\partial v/\partial x+\partial u/\partial y). \tag{13}$$

(3) v is continuous at $y=\pm f$.

$$v_1=v_0 \quad\text{at}\quad y=-f.$$
$$v_1=v_2 \quad\text{at}\quad y=+f. \tag{14}$$

The solution is required to be of the form $f(x, y)e^{i\omega t}$ with ω the same in plate and water. The variables are assumed separable, the separation constants in y being $-\eta_1^2$, $-\alpha_1^2$, $-\beta_1^2$, $-\eta_1^2$ for φ_0, φ_1, ψ_1, φ_2, respectively. The separation constant $-\xi^2$ for x must be the same for all φ's and ψ's for the desired wave amplitudes to be independent of x. This same requirement also specifies only one sign for ξ, either $+$ or $-$, i.e., that waves travel in only one direction in x along the plate. Then it can be easily verified that the solutions can be written in the form (all progressive waves),

$$\varphi_0=A_0\exp[i(\omega t+\xi x+\eta_1 y)],$$
$$\varphi_1=A_d'\exp[i(\omega t+\xi x+\alpha_1 y)]+B_d'\exp[i(\omega t+\xi x-\alpha_1 y)],$$
$$\psi_1=A_r'\exp[i(\omega t+\xi x+\beta_1 y)]+B_r'\exp[i(\omega t+\xi x-\beta_1 y)], \tag{15}$$
$$\varphi_2=A_2\exp[i(\omega t+\xi x-\eta_1 y)].$$

The separation constants are related to ω as

follows:

$$\xi^2+\eta_1{}^2=\frac{\omega^2}{c_w{}^2},\quad \xi^2+\alpha_1{}^2=\frac{\omega^2}{c_d{}^2},\quad \xi^2+\beta_1{}^2=\frac{\omega^2}{c_r{}^2}. \tag{16}$$

In this form only the indicated choices of sign are allowed for $i\eta_1$, to insure either an outgoing wave from the plate for $i\eta_1$ imaginary, or finiteness at infinity, for $i\eta_1$ real.

Equations (15) can be rewritten, for purposes of convenience, in the following form (standing waves in y only),

$$\begin{aligned}
&\varphi_0=A_0e^{+\eta y}e^{i(\omega t+\xi x)};\\
&\varphi_1=(A_d\cosh\alpha y+B_d\sinh\alpha y)e^{i(\omega t+\xi x)};\\
&\psi_1=(A_r\cosh\beta y+B_r\sinh\beta y)e^{i(\omega t+\xi x)};\\
&\varphi_2=A_2e^{-\eta y}e^{i(\omega t+\xi x)},
\end{aligned} \tag{17}$$

where

$$\xi^2-\eta^2=\frac{\omega^2}{c_w{}^2},\quad \xi^2-\alpha^2=\frac{\omega^2}{c_d{}^2},\quad \xi^2-\beta^2=\frac{\omega^2}{c_r{}^2}. \tag{18}$$

If the Eqs. (17) are substituted into the boundary conditions (12)–(14), the six Eqs. (19) are obtained. The common factor $e^{i(\omega t+\xi x)}$ has been cancelled from them all, and the common amplitude factor of each column is indicated at the top.

For a solution for the A's and B's to exist, the determinant of their coefficients must vanish. This determinant, for a given ω, determines the allowed values of ξ, and hence of α, β, η. There is in general an infinite set of discrete values for ξ, α, β, η for any given ω. They may be real or complex.

The determinant of the coefficients in (19) can be rearranged as is indicated in (20).

$$\begin{array}{cccccccc}
A_0 & A_d & A_r & B_d & B_r & A_2 & & \\
0 & \alpha\sinh\alpha f & -i\xi\cosh\beta f & +\alpha\cosh\alpha f & -i\xi\sinh\beta f & +\eta e^{-\eta f} & =0, & \\
-\eta e^{-\eta f} & -\alpha\sinh\alpha f & -i\xi\cosh\beta f & +\alpha\cosh\alpha f & +i\xi\sinh\beta f & 0 & =0, & \\
0 & +\mu_s(\xi^2+\beta^2)\cosh\alpha f & -\mu_s 2i\xi\beta\sinh\beta f & +\mu_s(\xi^2+\beta^2)\sinh\alpha f & -\mu_s 2i\xi\beta\cosh\beta f & -\lambda_w(\eta^2-\xi^2)e^{-\eta f} & =0, & (19)\\
-\lambda_w(\eta^2-\xi^2)e^{-\eta f} & +\mu_s(\xi^2+\beta^2)\cosh\alpha f & +\mu_s 2i\xi\beta\sinh\beta f & -\mu_s(\xi^2+\beta^2)\sinh\alpha f & -\mu_s 2i\xi\beta\cosh\beta f & 0 & =0, & \\
0 & +\mu_s 2i\xi\alpha\sinh\alpha f & +\mu_s(\xi^2+\beta^2)\cosh\beta f & +\mu_s 2i\xi\alpha\cosh\alpha f & +\mu_s(\xi^2+\beta^2)\sinh\beta f & 0 & =0, & \\
0 & -\mu_s 2i\xi\alpha\sinh\alpha f & +\mu_s(\xi^2+\beta^2)\cosh\beta f & +\mu_s 2i\xi\alpha\cosh\alpha f & -\mu_s(\xi^2+\beta^2)\sinh\beta f & 0 & =0, &
\end{array}$$

$$\begin{vmatrix}
\eta e^{-\eta f} & \alpha\sinh\alpha f & -i\xi\sinh\beta f & 0 & 0 & 0\\
\frac{-\lambda_w}{\mu_s}(\eta^2-\xi^2)e^{-\eta f} & (\xi^2+\beta^2)\cosh\beta f & -2i\xi\beta\cosh\beta f & 0 & 0 & 0\\
0 & 2i\xi\alpha\sinh\alpha f & (\xi^2+\beta^2)\sinh\beta f & 0 & 0 & 0\\
0 & 0 & 0 & \eta e^{-\eta f} & \alpha\cosh\alpha f & -i\xi\cosh\beta f\\
0 & 0 & 0 & \frac{-\lambda_w}{\mu_s}(\eta^2-\xi^2)e^{-\eta f} & (\xi^2+\beta^2)\sinh\alpha f & -2i\xi\beta\sinh\beta f\\
0 & 0 & 0 & 0 & 2i\xi\alpha\cosh\alpha f & (\xi^2+\beta^2)\cosh\beta f
\end{vmatrix}=0. \tag{20}$$

This form is more suitable for determining the various roots ξ_i, α_i, β_i, η_i for a given ω. In this form it is evident that either the upper or lower diagonal minor can be set equal to zero, as a condition that the entire determinant shall vanish.

This division of the sixth-order determinant into the product of two third-order determinants has a simple physical interpretation. It is possible to satisfy all six boundary conditions using four waves, with only three different amplitudes. One can use

$$\begin{aligned}
&\varphi_0=A_0e^{+\eta y}e^{i(\omega t+\xi x)},\\
&\varphi_1=A_d\cosh\alpha y e^{i(\omega t+\xi x)},\\
&\psi_1=B_r\sinh\beta y e^{i(\omega t+\xi x)},\\
&\varphi_2=A_2e^{-\theta y}e^{i(\omega t+\xi x)},
\end{aligned} \tag{21}$$

provided $A_2=A_0$.

These functions substituted into the boundary conditions give the upper diagonal minor of Eq. (20) as a condition for the existence of a solution. These four functions describe a motion of the plate symmetric about its median plane.

Alternatively, one can use

$$\begin{aligned}\varphi_0&=A_0e^{+\eta y}e^{i(\omega t+\xi x)},\\ \varphi_1&=B_d\sinh\alpha y e^{i(\omega t+\xi x)},\\ \psi_1&=A_r\cosh\beta y e^{i(\omega t+\xi x)},\\ \varphi_2&=A_2e^{-\eta y}e^{i(\omega t+\xi x)},\end{aligned}\tag{22}$$

provided $A_0=-A_2$. These four functions can also satisfy all six boundary conditions, and lead to the lower diagonal minor of (20). They describe a motion of the plate antisymmetric about its median plane. In the case of wave-lengths in x, $2\pi/\xi$, very much greater than the thickness of the plate, the symmetric and antisymmetric functions describe the so-called longitudinal and flexural vibrations of the plate, respectively.

One can therefore divide the discussion of the roots of the complete sixth-order determinant into separate discussions of the roots of the two third-order determinants. When any one amplitude A or B is given, the rest are determined by solving the set of five inhomogeneous equations obtained by dropping one of the six. Depending on whether the determinant for symmetric or antisymmetric motion has been set equal to zero, $A_0=\pm A_2$ and either $B_d=A_r=0$, or $A_d=B_r=0$.

Before proceeding with the numerical evaluation of the roots of these determinants for given ω, it would be well to point out some of the properties of these determinants, and their physical interpretation, if any.

Either determinant defines an implicit relation between *any two* of the following variables: ω, ξ, c_p, α, β, η. c_p is the phase velocity in the x direction, defined as ω/ξ. For a fixed value (real or complex) of any one variable there is an infinite set of values for the other. As one varies continuously, the other defines an infinite family of functions.

Those values of ξ real, or nearly real, for given ω real, correspond to modes of vibration propagated along the plate with no or little attenuation. If a ξ_i has an appreciable imaginary component, this mode is highly damped. The energy associated with it may be said to leak off rapidly into the surrounding medium. In case a transient disturbance, or superposition of modes, is propagated along the plate, after a relatively short distance of travel, only those modes with relatively small attenuation are significant, in summing to describe the shape of the transient.

The determinants for symmetric and antisymmetric motion in Eq. (20) are closely related to Lamb's equations for determining the symmetric and antisymmetric modes of a plate in a vacuum. The minors in Eq. (20) can be written, using the definitions in Eq. (11):

Symmetric.

$$\begin{aligned}&[(\xi^2+\beta^2)^2\cosh\alpha f\sinh\beta f\\ &\qquad-4\xi^2\alpha\beta\cosh\beta f\sinh\alpha f]+\left[\frac{\rho_w{c_w}^2}{\rho_s{c_r}^2}\right.\\ &\left.\times\frac{\eta^2-\xi^2}{\eta}\alpha(\beta^2-\xi^2)\sinh\beta f\sinh\alpha f\right]=0.\end{aligned}\tag{23}$$

Antisymmetric.

$$\begin{aligned}&[(\xi^2+\beta^2)^2\sinh\alpha f\cosh\beta f\\ &\qquad-4\xi^2\alpha\beta\sinh\beta f\cosh\alpha f]\\ &\qquad+\left[\frac{\rho_w{c_w}^2}{\rho_s{c_r}^2}\frac{\eta^2-\xi^2}{\eta}\alpha(\beta^2-\xi^2)\right.\\ &\qquad\left.\times\cosh\beta f\cosh\alpha f\right]=0.\end{aligned}\tag{24}$$

If these equations are compared to the corresponding ones given by Lamb for a plate in a vacuum (reference 2, Eqs. (12) and (48)), it will be seen that the first terms in Eqs. (23) and (24) above are just his Eqs. (12) and (48). The second terms are corrections introduced by the water. If the density or velocity of sound in water, in the second term, is made to approach zero, the second term vanishes and Eqs. (23) and (24) reduce to those derived by Lamb.

Now if the second term is small compared to the first, one can expect that solutions to the complete Eqs. (23) and (24) will be found close to those for the first term alone equal zero, i.e., close to the solutions already found by Lamb. The second term is small compared to the first, since $(\rho_w{c_w}^2/\rho_s{c_s}^2\eta)$ is small, except where $\eta=(\xi^2-\omega^2/{c_w}^2)^{\frac{1}{2}}$ is small. If Eqs. (23) and (24)

are multiplied by η, it will be seen that the smallness of η introduces additional solutions not in the neighborhood of those given by Lamb's solutions.

The equation for the symmetric case (23) is identical with the one for the antisymmetric case (24) except that hyperbolic sines in one replace hyperbolic cosines in the other, and *vice versa*. Therefore when the real parts of all the hyperbolic functions are large, the two expressions become practically identical. This is the case when the wave-length in x is small compared to the thickness of the plate, or the problem approaches the case of a semi-infinite elastic solid. There is then no distinction between symmetric and antisymmetric motion.

There are solutions at $\beta=0$ for all ω for the symmetric case, and at $\alpha=0$ for all ω for the antisymmetric case. Physically, this corresponds to the fact that at any excitation frequency some sort of disturbance, though very small, must initially be propagated down the plate with velocities c_r or c_d.

For purposes of computation it will be useful to have Eqs. (23) and (24) in dimensionless form. In terms of ξ, c_p they become:

Symmetric Case.

$$4\left[\left(1-\frac{1}{2}\frac{c_p^2}{c_r^2}\right)^2 \coth\left(\xi f\left(1-\frac{c_p^2}{c_d^2}\right)^{\frac{1}{2}}\right)\right.$$
$$\left.-\left(1-\frac{c_p^2}{c_d^2}\right)^{\frac{1}{2}}\left(1-\frac{c_p^2}{c_r^2}\right)^{\frac{1}{2}} \coth\left(\xi f\left(1-\frac{c_p^2}{c_r^2}\right)^{\frac{1}{2}}\right)\right]$$
$$+\frac{\rho_w c_w^2}{\rho_s c_r^2}\frac{1}{\left(1-\frac{c_p^2}{c_w^2}\right)^{\frac{1}{2}}}\frac{c_p^4}{c_w^2 c_r^2}\left(1-\frac{c_p^2}{c_d^2}\right)^{\frac{1}{2}}=0. \quad (25)$$

Antisymmetric Case.

$$4\left[\left(1-\frac{1}{2}\frac{c_p^2}{c_r^2}\right)^2 \tanh\left(\xi f\left(1-\frac{c_p^2}{c_d^2}\right)^{\frac{1}{2}}\right)\right.$$
$$\left.-\left(1-\frac{c_p^2}{c_d^2}\right)^{\frac{1}{2}}\left(1-\frac{c_p^2}{c_r^2}\right) \tanh\left(\xi f\left(1-\frac{c_p^2}{c_r^2}\right)^{\frac{1}{2}}\right)\right]$$
$$+\frac{\rho_w c_w^2}{\rho_s c_r^2}\frac{1}{\left(1-\frac{c_p^2}{c_w^2}\right)^{\frac{1}{2}}}\frac{c_p^4}{c_w^2 c_r^2}\left(1-\frac{c_p^2}{c_d^2}\right)^{\frac{1}{2}}=0. \quad (26)$$

Evidently when $c_p \simeq c_w$ both terms become of the same order of magnitude, so that there must be solutions for c_p in this neighborhood, almost independent of ω or ξ.

If for c_p one inserts ω/ξ in the above equations, and then holds ω fixed, it is evident that there must be solutions for ξ near $\xi=(2n+1)\pi i/f$ for the symmetric case, and near $\xi=n\pi i/f$ for the antisymmetric case. Both terms in Eqs. (25) and (26) become small under these conditions for n sufficiently large. These are the higher modes, and evidently they are very highly attenuated since ξ is almost purely imaginary.

As previously stated, if ω is considered as a continuous variable, there are families of functions $\xi_i(\omega)$, $c_{pi}(\omega)$, etc., defined by Eqs. (25) and (26). Following Lamb, the lower and more important of these functions will be evaluated for ω as a purely real variable, in particular the principal phase velocity curves $c_{pi}(\omega)$. Mathematically one finds where the contour of the real axis in the ω plane falls in the complex c_p plane, the mapping being defined implicitly by means of Eqs. (23) and (24) or their equivalents (25) and (26). There are infinitely many Riemann surfaces, of which only the more important ones are selected. This information is needed in order to determine the behavior of a transient as it progresses along the plate.

V. EVALUATION OF THE PHASE VELOCITY FOR VERY LARGE AND VERY SMALL WAVE-LENGTHS

Branches Close to Lamb's Curves. Long Wave-Length

For small values of ξf (wave-length large compared to the thickness of the plate) the hyperbolic sines and cosines can be replaced by linear terms or unity. The phase velocity for the principal symmetric mode is in this case

Branch B_1.

$$c_p=c_r\left[4\left(1-\frac{c_r^2}{c_d^2}\right)\right]^{\frac{1}{2}}\left[1+\frac{i}{2}\frac{\rho_w c_w^2}{\rho_s c_r^2}\frac{\omega f}{c_w}\right.$$
$$\left.\times\left(\frac{1}{4(1-c_r^2/c_d^2)}-\frac{c_r^2}{c_d^2}\right)\right]. \quad (27)$$

Lamb gives, under the same conditions for a

plate in a vacuum (reference 1, Eq. (15)), just the above expression with $\rho_w = c_w = 0$. This is the velocity of "longitudinal" waves in an elastic plate.

Evidently the effect of the water on the propagation of long longitudinal waves is not to change the real part of the phase velocity at all, but to add a very small attenuation which, for low frequencies, increases as the frequency.

For long wave-lengths, the determinant for the antisymmetrical modes gives for the phase velocity of long "flexural" waves,

$$c_p^2 = c_r^2\left[\frac{4}{3}f^3\xi^3\left(1-\frac{c_r^2}{c_d^2}\right)\frac{\rho_s}{\rho_w}\right]\frac{1}{1+\xi f\dfrac{\rho_s}{\rho_w}}. \qquad (28)$$

Here third-order terms must be included in expanding the hyperbolic functions, since the entire determinant vanishes in the first and second order.

Lamb's expression for the same condition with a plate in a vacuum is (reference 2, Eq. (50)),

$$c_p^2 = c_r^2\left[\frac{4}{3}f^2\xi^2\left(1-\frac{c_r^2}{c_d^2}\right)\right] \qquad (29)$$

which is the limit of (28) as $\rho_w \to 0$. Note however that with ξf very small and ρ_w finite, $c_p^2 \propto \xi^3 f^3$ for a plate in water, whereas $c_p^2 \propto \xi^2 f^2$ for a plate in a vacuum. Consider this limiting form,

$$c_p^2 = \frac{4}{3}c_r^2\left(1-\frac{c_r^2}{c_d^2}\right)\frac{\rho_s}{\rho_w}\xi^3 f^3, \qquad (30)$$

and change to variables c_p and ω. Then,

Branch B_4

$$c_p = \left[\frac{4}{3}c_r^2\left(1-\frac{c_r^2}{c_d^2}\right)\frac{\rho_s}{\rho_w}f^3\right]^{1/5}\omega^{3/5}$$

$$= \text{Const. } \omega^{3/5} = \text{Const. } |\omega|^{3/5}e^{i0}. \qquad (31)$$

Evidently if (31) satisfies (24) in the limit of ω small and pure real, with c_p real, then (24) is also satisfied by ω small and real with c_p complex. Equation (31) indicates that there are five different Riemann surfaces or branches from which a point of ω small and pure real in the ω plane can be mapped into the c_p plane. Of these five only Eq. (31) and

Branch B_2

$$c_p = \text{const } (|\omega|e^{4\pi i})^{3/5}$$

$$= \text{const } |\omega|^{3/5}e^{i(4/5)\pi/2}, \qquad (32)$$

are acceptable physically. The other three possible choices of angle for c_p, corresponding to 2π, 6π, 8π for the angle of ω, would indicate either propagation in the opposite direction, or negative attenuation.

There are therefore two acceptable phase velocities for long flexural waves represented in absolute magnitude by the same formula. One is highly attenuated, the other is not attenuated at all.

Branches Close to Lamb's Curves. Short Wave-Length

The determinants for symmetric and antisymmetric motion approach the same limiting form for short wave-length, or ξf large.

$$[(\xi^2+\beta^2)-4\xi^2\alpha\beta]+\frac{\rho_w c_w^2}{\rho_s c_r^2}\frac{\eta^2-\xi^2}{\eta}\alpha(\beta^2-\xi^2)=0. \qquad (33)$$

This is the same expression as that found by Lamb for a plate in a vacuum, again plus a small correction term due to the water. Lamb's expression was the same as that deduced by Rayleigh for seismic waves on the surface of the earth. The above expression therefore corresponds to Rayleigh waves on the floor of the ocean.

The phase velocity can be deduced from (33) by assuming that $c_p = R(1+\epsilon)$, $\epsilon \ll 1$, where R is the velocity of Rayleigh waves, determined by setting the first term alone of Eq. (33) equal to zero. If $c_p = R(1+\epsilon)$ is substituted into Eq. (33), or the equivalent expressions (25) or (26) with the hyperbolic functions set equal to unity, one can evaluate ϵ. It is assumed that the second, water correction term is small. In this way one finds

Branches B_1, B_2

$$c_p = R(1+\epsilon), \qquad (34)$$

where

$$R = 0.9194c_r = 0.303 \cdot 10^6 \text{ cm/sec.} \qquad (35)$$

$$\epsilon=-i\frac{\dfrac{\rho_w c_w^2}{\rho_s c_r^2}\dfrac{R^4}{c_w^2 c_r^2}\left(1-\dfrac{R^2}{c_d^2}\right)^{\frac{1}{2}}}{4\left(\dfrac{R^2}{c_w^2}-1\right)^{\frac{1}{2}}\left[2\dfrac{R^2}{c_r^2}\left(1-\dfrac{1}{2}\dfrac{R^2}{c_r^2}\right)-\dfrac{R^2}{c_d^2}\left(\dfrac{1-\dfrac{R^2}{c_r^2}}{1-\dfrac{R^2}{c_d^2}}\right)^{\frac{1}{2}}-\dfrac{R^2}{c_r^2}\left(\dfrac{1-\dfrac{R^2}{c_d^2}}{1-\dfrac{R^2}{c_r^2}}\right)^{\frac{1}{2}}\right]}. \tag{36}$$

The numerical value for ϵ, using the data in Eq. (11) is

$$\epsilon=i1.18\cdot10^{-2}. \tag{37}$$

Thus the attenuation of Rayleigh waves on a steel-bottomed ocean would be relatively small, so far as losses by acoustic radiation are concerned. In practice, since the wave-length of seismic waves is comparable to the depth of the ocean, the finite depth of the ocean and gravity waves on its surface would have to be taken into account.

Branches Not Close to Lamb's Curves

The form of Eqs. (25) and (26), multiplied by $(1-c_p^2/c_w^2)$, indicates that branches of the phase-velocity curve may be found for c_p real and less than but almost equal to c_w. It can be shown that the first and second terms of Eqs. (24) and (25) are of opposite sign, so that they can be made to cancel. For c_p real, greater than but almost equal to c_w, the first term is imaginary, the second real, hence their sum can never vanish.

For ξf small, or long wave-lengths, the determinant of the symmetric case may be approximated to give

Branch B_3

$$\frac{c_p}{c_w}=1-\frac{\left[\dfrac{\rho_w c_w^2}{\rho_s c_r^2}\left(1-\dfrac{c_w^2}{c_d^2}\right)\dfrac{f\omega}{c_w}\right]^2}{32\left[1-\dfrac{1}{4}\dfrac{c_w^2}{c_r^2}+\dfrac{c_r^2}{c_d^2}\right]}$$

$$=1-1.25\cdot10^{-5}\frac{f^2\omega^2}{c_w^2},\quad\left(\frac{f\omega}{c_w}<1\right). \tag{38}$$

For ξf small, c_p in the determinant for the antisymmetrical case does not remain close to c_w but approaches zero in one of the two ways indicated in Eqs. (31) and (32). In fact, examination of the original Eqs. (24) or (26) will show that as $\omega\to0$, $\xi\to0$ and always in such a way that $\omega/\xi=c_p\to0$.

For ξf large, or short wave-lengths, the antisymmetric and symmetric forms approach each other, and if c_p is suspected to be real and less than c_w, the equation which must be satisfied is

$$\left(1-\frac{c_p^2}{c_w^2}\right)^{\frac{1}{2}}\left[4\left(1-\frac{1}{2}\frac{c_p^2}{c_r^2}\right)^2-4\left(1-\frac{c_p^2}{c_d^2}\right)^{\frac{1}{2}}\times\left(1-\frac{c_p^2}{c_r^2}\right)^{\frac{1}{2}}\right]+\frac{\rho_w c_p^4}{\rho_s c_r^4}\left(1-\frac{c_p^2}{c_d^2}\right)^{\frac{1}{2}}=0. \tag{39}$$

These two terms are of opposite sign, so that $c_p<c_w$ and real is a possibility. Substituting $c_p=c_w$ in all terms except in the factor $(1-c_p^2/c_w^2)^{\frac{1}{2}}$, one can obtain the approximate expression

Branches B_3, B_4

$$\frac{c_p^2}{c_w^2}=1-\left[\frac{\dfrac{\rho_w c_w^4}{\rho_s c_r^4}\left(1-\dfrac{c_w^2}{c_d^2}\right)^{\frac{1}{2}}}{4\left[\left(1-\dfrac{c_w^2}{c_d^2}\right)^{\frac{1}{2}}\left(1-\dfrac{c_w^2}{c_r^2}\right)^{\frac{1}{2}}-\left(1-\dfrac{1}{2}\dfrac{c_w^2}{c_r^2}\right)^2\right]}\right]^2. \tag{40}$$

Using the numerical values in Eq. (11),

$$c_p/c_w=1-0.00019. \tag{41}$$

In summary therefore, there is one symmetric and one antisymmetric branch of the phase velocity curve close to c_w. They are both the same constant fraction of c_w at high frequencies and real. At low frequencies the symmetric one approaches c_w quadratically with respect to frequency, and is real. The antisymmetric one must diverge from c_w and approach zero. It is also real, and was shown numerically to go into Eq. (31).

VI. EVALUATION OF THE PHASE VELOCITY FOR INTERMEDIATE WAVE-LENGTHS. CONNECTION OF THE BRANCHES

Thus far the extremes only of the phase velocity *vs.* frequency curves have been evaluated,

and compared with the results of Lamb for a plate in a vacuum. The various branches so found are summarized in Fig. 3, Lamb's curves, and Fig. 4, limiting curves for high and low frequency for a plate in water. A beaded line indicates that

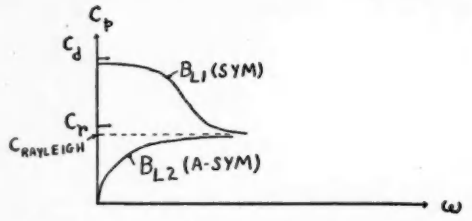


FIG. 3. Phase velocity as a function of circular frequency for a plate in a vacuum, according to Lamb.

the phase velocity has a small imaginary component (slight attenuation), an open circle line indicates a large imaginary component (large attenuation), and a full line indicates that the phase velocity is all real (no attenuation). Full horizontal lines at c_d and c_r should appear on both sketches, but have been omitted to avoid confusion.

The question now arises, where do the intermediate points on these branches lie? Also, what branches on the left in Fig. 4 tie in with what branches on the right?

To answer these questions, consider the Eqs. (24) and (25) as giving a functional relation between two complex variables implicitly. Choose for example c_p and ω. Either (24) or (25) can be written

$$F(c_p, \omega) = R(c_{pr}, c_{pi}, \omega_r, \omega_i) + iI(c_{pr}, c_{pi}, \omega_r, \omega_i) = 0, \quad (42)$$

where c_{pr}, c_{pi}, ω_r, ω_i are the real and imaginary parts of c_p and ω.

There are therefore three complex planes, the F plane, the c_p plane, and the ω plane. Imagine that a point be fixed in the ω plane, and a point in the c_p plane describes a closed curve. The corresponding point in the F plane will also describe a closed curve, provided that $F(c_p, \omega)$ is regular over the range of variables considered. If the closed curve in the F plane incloses the origin, it means that a point on the implicit relation between c_p and ω, given by $F(c_p, \omega) = 0$ has been located. It lies somewhere within the closed curve in the c_p plane, for the given point ω.

This process was used to locate points in the intermediate regions of Fig. 4, establish the connections between the right and left branches, and also as an independent numerical check on the analytic derivations of Eqs. (27), (28), (36), (38), and (41). The procedure is as follows. A point ω_0, c_{p0} is chosen either on one of Lamb's curves, or in some other region where a solution is suspected, such as c_p close to c_w. The values ω_0, $c_{p0}(1+\epsilon e^{i\theta})$ are then substituted into Eq. (25) or (26), and each factor of each term expanded by Taylor's theorem. ϵ is assumed small compared to unity. In some cases, for example when c_p is near c_w, ϵ must be very small indeed. If the choice ω_0, c_{p0} was a good guess, the value of the determinant, Eq. (25) or (26) will become very small, and ϵ and θ can then be adjusted to ϵ_0, θ_0 to make the entire expression vanish. When a value for ϵ_0 has been so determined, one must subsequently check that the expansions used to obtain it were valid. A point on the implicit relation between ω and c_p, ω_0 and $c_{p0}(1+\epsilon_0 e^{i\theta_0})$ has thus been determined.

In the case of Branch B_2, a method similar to that used to obtain Eq. (36) will give intermediate points on the phase-velocity curve. If one lets $F(\omega, c_p)$ stand for Eq. (25), the symmetric case, then

$$F(\omega, c_p) = \Delta_L(\omega, c_p) + c(\omega, c_p) = 0, \quad (43)$$

where Δ_L is Lamb's determinant for the plate in a vacuum and $c(\omega, c_p)$ is the small correction term owing to the water. Let ω_0, c_{p0} be points on Lamb's phase-velocity curve $B_{L1}(\Delta_L(\omega_0, c_{p0}) = 0)$, and substitute ω_0, $c_{p0}(1+\epsilon)$ into Eq. (42).

$$F(\omega_0, c_{p0}(1+\epsilon)) = \Delta_L(\omega_0, c_{p0}) + \frac{\partial \Delta}{\partial c_p}(\omega_0, c_{p0})\epsilon c_{p0} + c(\omega_0, c_{p0}) = 0,$$

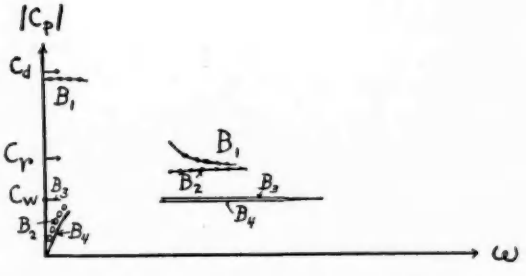


FIG. 4. Limiting values of absolute magnitude of phase velocity, as a function of circular frequency ω, for a plate in water.

or

$$\epsilon = \frac{-c(\omega_0, c_{p0})}{(\partial \Delta_L/\partial c_{p0}) \cdot c_{p0}}. \tag{44}$$

Since $\partial \Delta_L/\partial \omega$ is easier to use than $\partial \Delta_L/\partial c_p$, one

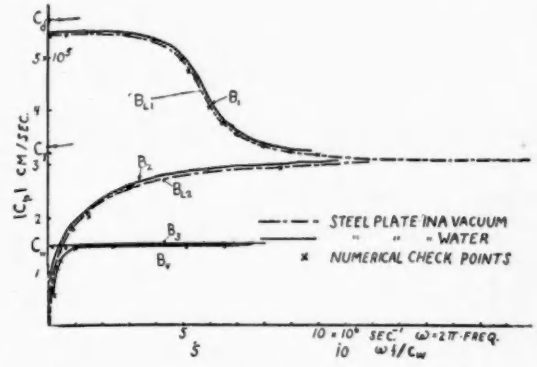


FIG. 5. Principal phase-velocity curves of a steel plate in air and water.

can also write

$$\epsilon = \frac{+c(\omega_0, c_{p0})}{c_{p0} \dfrac{\partial \Delta_L}{\partial \omega} \left(\dfrac{d\omega_0}{dc_{p0}} \right)_{B_{L1}}}, \tag{45}$$

where $(d\omega_0/dc_{p0})_{B_{L1}}$ is taken graphically from Lamb's phase-velocity curve for B_{L1}, in Fig. 3 or Fig. 5. Since for $c_{p0} > c_w$ and real the correction term is pure imaginary, Eq. (45) leads to the conclusion that the water around the plate does not change the real part of the phase velocity, but only adds a small attenuation. Also, barring unusual behavior of $c(\omega_0, c_{p0})$ and $\partial \Delta/\partial \omega$, this attenuation should be a maximum where $(d\omega_0/dc_{p0})_{B_{L1}}$ is a minimum. This is at the intermediate frequencies, where the wave-length is neither large nor small compared to the thickness. This expectation was borne out by the numerical calculations (Fig. 6).

If this process is applied to determining the antisymmetrical Branch B_2, close to B_{L2}, it fails as $c_p \to c_w$, but in so doing indicates that the imaginary component of the phase velocity becomes very large. For this reason it was inferred that at low frequencies B_2 connected with the highly attenuated phase-velocity curve whose analytic expression is given in Eq. (32).

In this way the connections required for Fig. 4 were completed, and are indicated in Fig. 5. Crosses (x) in Figs. 5–9 indicate numerical values computed by one of the above methods. In Fig. 5 the absolute magnitudes of all the phase velocities are plotted as a function of the circular frequency ω. The separation of B_1 and B_{L1}, B_2 and B_{L2} is somewhat exaggerated. In Figs. 6–9 the detailed dependence of the individual branches on the dimensionless variable $\omega f/c_w$ is given, so that these curves can be applied to a plate of any thickness. The solid lines indicate analytic curves, and the dotted lines the interpolated guess. For Branches B_1 and B_2, Figs. 6 and 7, the imaginary component of the phase velocity c_p is plotted against the real component, with $\omega f/c_w$ as parameter for the check points. For Branches B_3 and B_4, Figs. 8 and 9, which are all real and close to c_w, $1 - c_p/c_w$ was used as dependent variable, and $\omega f/c_w$ as independent variable.

In Fig. 5 the symmetric Branch B_1 followed quite closely the corresponding Lamb curve B_{L1} throughout. The imaginary component has a maximum at intermediate values of wave-length ($\omega f/c_w \sim 5$, Fig. 6). The irregularity of the points for large and small $\omega f/c_w$ reflects the difficulty of following the curve numerically in these regions.

In Fig. 5 the antisymmetric Branch B_2 clings closely to the Lamb Branch B_{L2} until it dips to the c_w level. It then diverges rapidly from the Lamb curve in that its imaginary component becomes very large. At low frequencies it has the same absolute magnitude dependence on ω as Branch B_4, and is close to B_{L2} in absolute magnitude, but the imaginary component is large (Fig. 7).

Physically, we may interpret this antisymmetric mode as a high pass filter, the cut-off point being determined approximately at that frequency where the phase velocity equals the velocity in the surrounding medium, water. Frequencies below this cut-off, where $c_p < c_w$ are highly attenuated, the energy being lost by radiation into the water. Above it, where $c_p > c_w$, they are only slightly attenuated.

In Figs. 5 and 8, the symmetric Branch B_3 is close to the velocity of sound in water throughout. At high frequencies it merges into B_4 at a constant fraction of c_w; at low frequencies it approaches c_w.

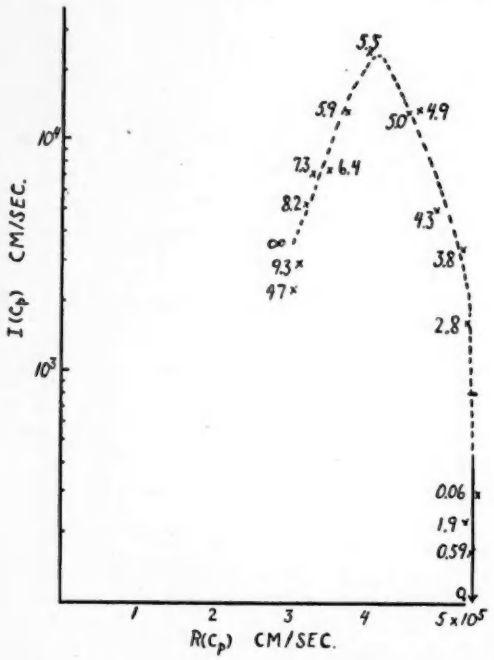


FIG. 6. Branch B_1. Imaginary *vs.* real part of phase velocity, with $\omega f/c_w$ as a parameter.

In Figs. 5 and 9, the antisymmetric Branch B_4, also just below c_w at high frequencies, follows the c_w level quite closely, almost to the point where it would cross the Lamb antisymmetric Branch B_{L2} (Fig. 5). It then turns abruptly down, and remains just below the Lamb branch B_{L2}. It is real throughout. It is very difficult to explore numerically the antisymmetric case for small ω, since all the terms are approaching zero. It was necessary in this region to carry out the analysis with the variables c_p and ξ, and subsequently convert to c_p and ω.

The dimensionless variable $\omega f/c_w$ used in Figs. 6–9 is the ratio of the time required for sound to traverse the half-thickness of the plate in water, to the time required for the exponent in the time-dependent factor $e^{i\omega t}$ in φ or ψ to change through one radian. It will be observed that, except for Branch B_1, when this ratio is of order unity most of the changes in the phase velocity curves are taking place. Branch B_3 is beginning to approach c_w (Fig. 8), B_4 is turning sharply down (Fig. 5), and B_2 is changing from predominantly real to predominantly imaginary (Fig. 7). The frequency corresponding to $\omega f/c_w \sim 1$ is 160 kc, period $\sim 6\mu$ sec. For Branch B_1 the change occurs around $\omega f/c_w \sim 5$, or at a frequency of approximately 800 kc, period $\sim 1.2\mu$ sec. A conversion scale from $\omega f/c_w$ to ω appears on Fig. 5.

VII. USE OF PHASE-VELOCITY CURVES IN THE STUDY OF TRANSIENTS

According to Lamb,[4] in a one-dimensional dispersive medium, the effect of an initial disturbance can be represented by an integral of the form

$$\varphi(x,t)=\int_0^\infty A(\xi)e^{i(\omega t+\xi x)}d\xi, \tag{46}$$

where ω as a function of ξ is determined by the phase velocity curve, which in this report is given as $c_p=\omega/\xi$ as a function of ω. $A(\xi)$ is the Fourier transform of the shape of the disturbance at $t=0$. For example, if $A(\xi)=$constant, the initial disturbance is infinitely concentrated, a δ function. If $A(\xi)=\pi\delta(\xi)-i/\xi$, the initial disturbance is a step, or unit function, $1(x)$.

For the problem of the elastic plate in water, Eq. (46) would be generalized to

$$\begin{aligned}
\varphi_0&=\sum_{i=1}^{\infty}\int_0^\infty A_{0i}(\xi)\exp(+\eta_i y)\\
&\qquad\times\exp[i(\omega_i t+\xi x)]d\xi,\\
\varphi_1&=\sum_{i=1}^{\infty}\Big[\int_0^\infty (A_{di}(\xi)\cosh\alpha_i y\\
&\qquad\times\exp[i(\omega_i t+\xi x)]+B_{di}(\xi)\sinh\alpha_i y\\
&\qquad\times\exp[i(\omega_i t+\xi x)])d\xi\Big],\\
\psi_1&=\sum_{i=1}^{\infty}\Big[\int_0^\infty (A_{ri}(\xi)\cosh\beta_i y\\
&\qquad\times\exp[i(\omega_i t+\xi x)]+B_{ri}(\xi)\sinh\beta_i y\\
&\qquad\times\exp[i(\omega_i t+\xi x)])d\xi\Big],\\
\varphi_2&=\sum_{i=1}^{\infty}\int_0^\infty A_{2i}(\xi)\exp(-\eta_i y)\\
&\qquad\times\exp[i(\omega_i t+\xi x)]d\xi.
\end{aligned} \tag{47}$$

In (47), ω_i, α_i, β_i, η_i are functions of ξ determined

[4] H. Lamb, *Hydrodynamics* (Cambridge University Press, New York, 1932), fourth edition, p. 396.

by the phase-velocity curves, and the summation is applied over all the branches, an infinite number of them, i.e., all the normal modes. The $A_i(\xi)$, $B_i(\xi)$ are so chosen that the configuration of the φ's and ψ's with respect to y and x is correctly given at the initial instant, $t=0$.

However the amplitudes $A_i(\xi)$, $B_i(\xi)$ of the higher modes are appreciable only when there is considerable "structure" or variation with respect to y in the initial disturbance, not true in the present problem. Also it was shown that these higher modes are damped out rapidly as the disturbance progresses across the plate. Therefore it will be considered sufficient to use the lowest members only, or branches of the phase velocity curve which have just been determined.

Even if the amplitudes $A_i(\xi)$, $B_i(\xi)$ could be determined there would still remain the difficult problem of carrying out the integration over ξ,

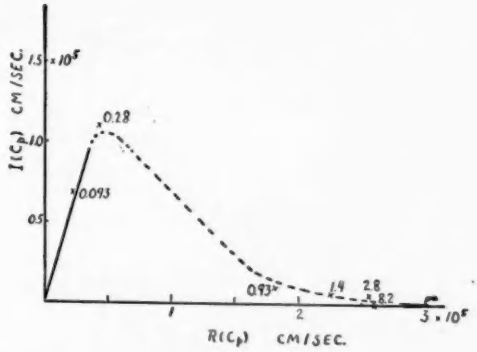


FIG. 7. Branch B_2. Imaginary *vs.* real part of phase velocity, with $\omega f/c_w$ as a parameter.

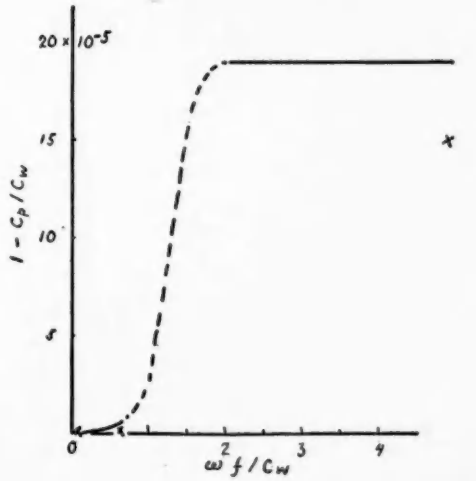


FIG. 8. Branch B_3. Fractional departure of phase velocity from velocity of sound in water, as a function of $\omega f/c_w$.

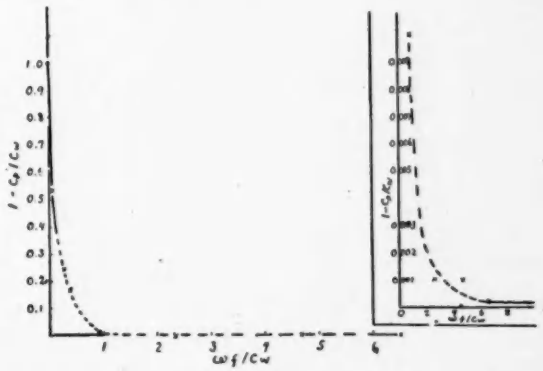


FIG. 9. Branch B_4. Fractional departure of phase velocity from velocity of sound in water, as a function of $\omega f/c_w$.

where the relations $\omega_i(\xi)$, $\alpha_i(\xi)$, etc. are defined implicitly and the variables are actually complex. The problem of contour integration of a function of a complex variable defined implicitly seems to have no known general solution. Here the path of integration is along the real axis, but it could be deformed. Lamb[4] gives an approximate method which will be modified slightly to apply to the present problem.

The oscillograms in Fig. 14, showing the precursor, represent observations of pressure as functions of t, for various values of x, with y fixed, as shown in Fig. 1. Therefore the problem is to evaluate, say from φ_2

$$(p_{xx})_{\varphi_2}=(p_{yy})_{\varphi_2}=\lambda_w\left(\frac{\partial^2\varphi_2}{\partial x^2}+\frac{\partial^2\varphi_2}{\partial y^2}\right)$$

$$=\lambda_w\sum_i\int_0^\infty(\eta_i^2-\xi^2)A_{2i}(\xi)$$

$$\times\exp(-\eta_i y)\exp[i(\omega_i t+\xi x)]d\xi, \quad (48)$$

as a function of t, with x as parameter. One would like to know what characteristic frequency or frequencies appear, that is the average "spacing of the crests"[4] which appear in the oscillogram. Assume that the factor $(\eta_i^2-\xi^2)\cdot A_{2i}(\xi)$ changes slowly by comparison to the factor $e^{i(\omega t+\xi x)}$. This will be true for a concentrated δ or step disturbance, and will also be true for an exponential-type explosion wave. Then the principal contribution to the integrand comes in those ranges of ξ where the exponent is stationary, since then all the elemental vectors represented by the integrand add in a straight line. The ω corresponding

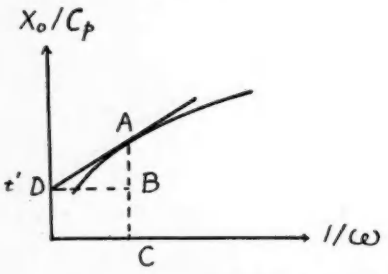


FIG. 10. Determination of characteristic oscillation frequency, for a pulse in a dispersive medium.

to this ξ is the one which appears predominantly in the oscillogram and determines the average spacing of the crests. The wave-length $2\pi/\xi$ would characterize a "snapshot" or instantaneous picture of the disturbance. The factor $e^{-\eta y}$ is neglected, since y is a constant. Hence this factor does not affect the relative value, with respect to x and t, of the integral.

To determine this characteristic ω a modification of the construction given by Lamb[4] is used. In Fig. 10 x_0/c_p is plotted against $1/\omega$, where x_0 is the x coordinate of the point of observation. The ordinate has therefore the dimension of time. At any particular time t', given by a point on the y axis, the characteristic oscillation frequencies are given by the abscissae of the points of tangency, such as C, of straight lines drawn from the point t' to the x_0/c_p curve. Let A be in the first instance any point, not necessarily the tangent, on the x_0/c_p curve. The angle ADB has as tangent AB/BD, which is

$$\left(\frac{x_0}{c_p}-t'\right)\Big/\frac{1}{\omega}=\omega\left(\frac{x_0}{c_p}-t'\right)=\xi x_0-\omega t' \quad (49)$$

and this tangent, hence the phase angle $\xi x-\omega t$, is stationary when the line DE is drawn tangent to the x_0/c_p curve. Evidently as time goes on the point D moves up the y axis, and the tangent line rolls on the x_0/c_p curve. If this curve is concave down, as drawn, $1/\omega$ increases, and the maxima in the oscillogram become farther and farther apart. The characteristic frequency decreases with time. When the x_0/c_p curve is concave up, the opposite is true. If the x_0/c_p curve has sharp bends in it, as time goes on and D moves up, the abscissa C of the characteristic frequency shifts only slightly. Thus over a considerable interval of time the spacing of the crests remains nearly the same, and this particular oscillation will appear very prominently in the oscillogram. In general in a dispersive medium, when the phase velocity *vs.* frequency curve changes rapidly, i.e., has bends, the frequency characteristic of the transition region will appear in a transient disturbance. The commonest example of this is given by electrical filters with sharp cut-offs (c_p becomes almost all imaginary) or abrupt changes in their transmission characteristics. When excited by a transient, these filters often "ring" with oscillations characteristic of the cut-off or transition regions.

For a bend in the x_0/c_p curve to be sharp enough to make the corresponding frequency in the oscillogram prominent, a shift of $\Delta t'$ at D, Fig. 10, equal to the period corresponding to C, P_C, must cause a shift in P_C small compared to P_C. In other words the bend must be so sharp that the characteristic period changes by a small fraction of itself over the period of a single oscillation.

$$dP_c/dt \ll 1. \quad (50)$$

If there is no dispersion in the medium, as in open water, the x_0/c_p curve is a straight line parallel to the $1/\omega$ axis. Then no tangent can be drawn until $t'=x_0/c_p$, when tangent and x_0/c_p curve coincide, or are tangent at all values of $1/\omega$. That is, the original undistorted disturbance, or at least an aperiodic disturbance, is observed.

The method of stationary phase described above applies primarily to cases where ω and ξ are all real. For complex values the saddle point method can be applied, which amounts to deforming the path of integration from the real

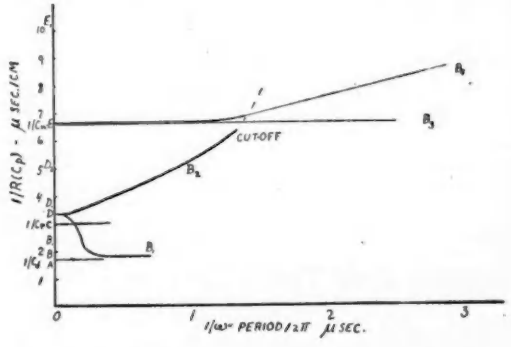


FIG. 11. Reciprocal of real part of phase velocity, *vs.* reciprocal of circular frequency.

axis (stationary phase method) into the complex plane. However, when the relation between ω and ξ is defined implicitly, this process usually becomes too complicated to be tractable, though a formal solution can be written down. In the present problem the stationary phase method will be used, using only the real part of c_p, and the imaginary part, or the attenuation, will be neglected. This is probably a good approximation for all the branches except the low frequency end of B_2, since every where else the imaginary part of c_p is indeed moderately small. This approximation is equivalent to assuming that all the elemental vectors $e^{i(\omega t+\xi x)}$ are of the same length. When the tangent falls on that part of B_2 with large attenuation, where the approximation is known to fail, the entire contribution from that branch will be neglected, on the grounds that those frequencies will be damped out.

This approximation is a mathematical necessity, for which the justification is perhaps a little meagre. As it turns out, the precursor can be associated with Branch B_4 which *is* all real, so that where the theory agrees with observation, the approximation is valid.

VIII. APPLICATION OF STATIONARY PHASE METHOD

These arguments can be applied to the present problem of the propagation of the explosion wave along a steel plate in water. In Figs. 11 and 12 is plotted $1/c_p$ *vs.* $1/\omega$ for all the phase-velocity curves evaluated. $1/c_p$ was chosen rather than x_0/c_p with a particular x_0, since the shape of the curves would be unaffected by including x_0. The origin in coordinates x and t is the edge of the plate, at the time the explosion wave strikes it. A point in time on the $1/c_p$ axis in Fig. 11, such as C, is at such a distance from the origin that a disturbance traveling with the velocity c_r would have just reached it. A disturbance traveling with a velocity c_d would have already gone by. It would have arrived at an earlier (smaller) time corresponding to the point A. Plate 6 can therefore be made to apply to any position of the gauge, by a proper choice of scale for the $1/c_p$ axis.

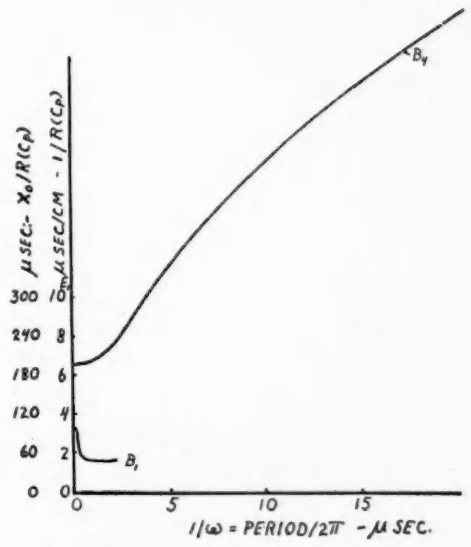


FIG. 12. Reciprocal of real part of phase velocity, *vs.* reciprocal of circular frequency. x_0 = radius of disk (30.5 cm).

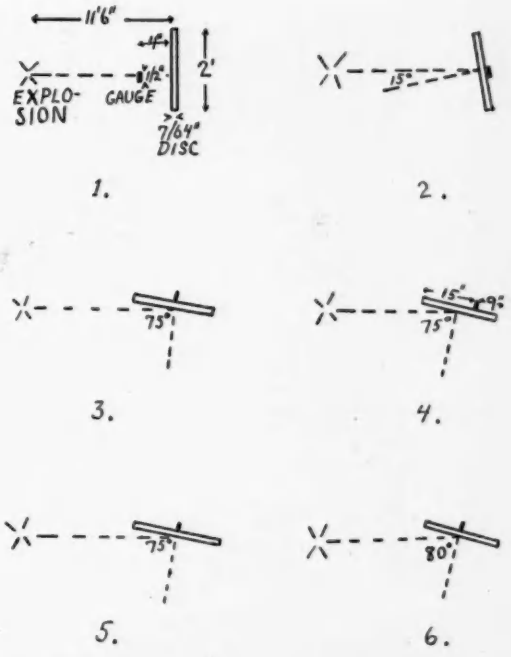


FIG. 13. Oscillograms of explosion waves in the neighborhood of a steel disk in water. (Legend to Fig. 14.) (1) Incident and reflected wave. Sweep duration 900μ sec. (2) Transmitted wave, showing postcursor. Sw. dur. 900μ sec. (3) Transmitted wave, showing precursor. Sw. dur. 175μ sec. (4) Same as (3). (5) Same as (3) but with Sw. dur. 900μ sec. (6) Same as (5). The explosion was from No. 8 Dupont blasting caps.

Consider for example the observed oscillograms when the gauge is at the middle of the disk, 30.5 cm from the edge. The explosion lies nearly in the plane of the disk, so that the explosion wave strikes the disk on its edge, as in Fig. 13, No. 3.

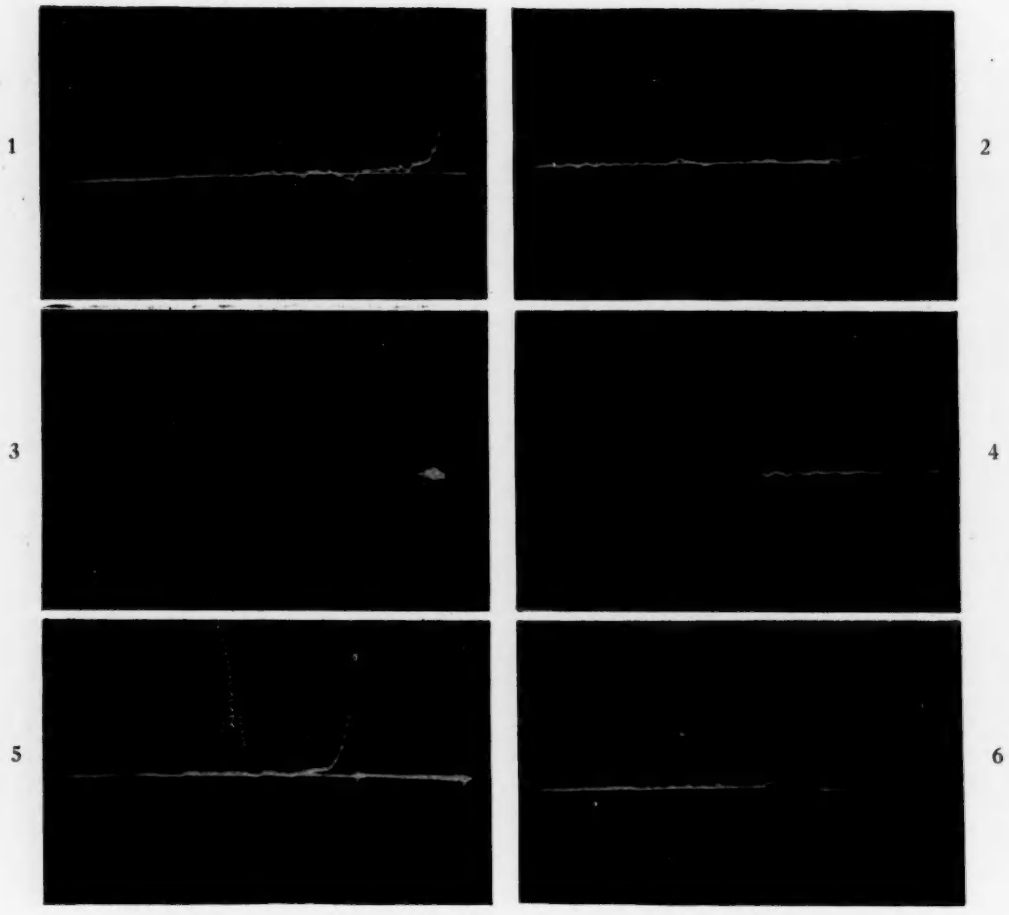


FIG. 14. Oscillograms of explosion waves in the neighborhood of a steel disk in water.

At $t=0$, the origin, Fig. 11, no tangents can be drawn, and there is no disturbance at the gauge. At time $t=x/c_d$ (A, Fig. 11), corresponding to the time required for a wave in the plate to reach the position of the gauge, traveling with a velocity c_d, a tangent to the straight line solution $1/c_p=1/c_d$ can be drawn. It is "tangent" at all frequencies. It does not necessarily follow that the resulting disturbance will be similar to the original disturbance, but merely that it will be aperiodic.[5]

Shortly after $t=x/c_d$ another aperiodic disturbance will arrive, corresponding to the tangent to the low frequency end of B_1 (Point B), or long longitudinal waves. Later two more aperiodic disturbances arrive (Points C, D), corresponding to the arrival of "underwater Rayleigh waves" on the surface of the plate, and the straight line solution $1/c_p=1/c_r$.

At intermediate and immediately subsequent times (Points B_1, D_1) tangents to the knees of Branch B_1 can be drawn, with characteristic oscillations ranging in period between 0.6 to 1.9μ sec., or $1/\omega$ from 0.1 to 0.3μ sec. These are too short to be easily observed with the oscillograph and sweep speed actually used.

Beginning at a time corresponding approximately to D_2, tangents can be drawn to Branch B_4. Since B_4 is concave up, as time goes on the tangent rolls toward the $1/c_p$ axis and the spacing of the crests in the oscillogram decreases, until at $t=x/c_w$ (Point E) another aperiodic disturbance arrives which has been traveling with a velocity equal to the velocity of sound in water. This is just the behavior of the precursor, which precedes the main explosion wave in the water. Much

[5] H. Lamb, Phil. Trans. Roy. Soc. London **A203**, 1 (1904).

later, at a time such as E_1 (see also Fig. 12), tangents which fulfill the condition for periodicity can be drawn to the parabolic, concave down part of Branch B_4. This characteristic oscillation has a decreasing frequency, or increasing spacing of the crests.

The procedure outlined above does not give any indication of the treatment accorded Branch B_2, which shifts rapidly from primarily real to primarily imaginary at about $1/\omega = 1.2\mu$ sec. If the analogy of B_2 to a high pass filter is a good one, it might be suspected that oscillations characteristic of the cut-off frequency should appear, when this mode is sufficiently excited by the transient. The tangent which can be drawn to the primarily real part of B_2, between times C and D, does not fulfill the condition for periodicity.

The observations, Fig. 14, made with a tourmaline pressure gauge and cathode-ray oscillograph, can be compared with the above description of the periodic and aperiodic disturbances arriving at the center of the disk. There are a number of discrepancies. None of the disturbances corresponding to $1/c_d$, $1/c_r$, and Branch B_1 appear. The absence of pulses corresponding to $1/c_d$ and $1/c_r$ must be attributed to smallness of the corresponding amplitude factors, $A_{c_d}(\xi)$ and $A_{c_r}(\xi)$. The theory is insufficient to determine these factors, but from a physical standpoint one would expect them to be small. They represent disturbances propagated in the plate as though it were an infinite medium. Actually it is quite thin, in terms of wave-lengths to which the measuring instrument is responsive. Other investigations of more favorable examples have shown that these amplitudes are relatively small.[5]

The tangent points to the knees and the Rayleigh-region end of B_1 fall at abscissae of high frequency, $\gtrsim$ one megacycle, which the oscillograms cannot resolve. The absence of a disturbance corresponding to the low frequency part of B_1 remains unexplained, unless one wishes to infer from the observations that the amplitude factor here is small. For very low frequencies the wave-length is comparable to the radius of the disk, and the theory is not applicable.

The same remarks also apply to Branch B_2. Here the low frequency end is highly attenuated. It may be that the cut-off frequency is contributing to the precursor.

The lower tangent to B_4 gives a very satisfactory explanation of the precursor. Theory and observation of the velocity, duration, number of oscillations, and their range of period can be compared in the following table. In Fig. 11, if D_2 is specified at 52μ sec./cm, the velocity of the precursor corresponds to D_2, the duration to $x/c_{D2} - x/c_w$, with $x = 1$ ft. (30.5 cm). The range in period of the characteristic oscillation corresponds to $1/\omega = 2\mu$ sec. to $1/\omega = 1\mu$ sec.

Properties of precursor.

	Theory	Observed (from II)
velocity	0.19×10^6 cm/sec.	0.21×10^6 cm/sec.
duration	40μ sec.	40–70μ sec.
range of period	12 to 6μ sec.	11 to 8.5μ sec.
number of oscillations	5	6

Also note that the duration of the precursor and hence the number of oscillations increase linearly with the scale of the y axis, or distance from the edge of the plate. This is in agreement with observation. The theory is insufficient to describe either the amplitude of the precursor as a function of time, nor the large negative peak often observed just before the main shock wave. A good example of this negative peak appears in Fig. 14, No. 6.

The above analysis applies to the case where the explosion wave lies nearly in the plane of the disk, at angles of incidence near 90°, as in Fig. 13, No. 3. Evidently if the explosion wave comes from a direction more nearly normal to the disk, as in Fig. 13, No. 2, it will arrive sooner at the gauge. Hence it will overlap the precursor, and precursor oscillations will appear after the arrival of the shock front. At an angle of incidence of about $\arccos(c_w/c_{\text{prec}}) = 57°$, precursor and explosion wave will arrive simultaneously, i.e., there is no "pre" cursor. This conclusion is also in approximate agreement with observation.

Theory requires that there should also be oscillations of period increasing with the time, corresponding to an upper tangent on the parabolic part of B_4, in Fig. 12. These oscillations are those given by the classical theory of a struck bar, or flat plate.[6] The periodicity condition

[6] Lord Rayleigh, *Theory of Sound* (Macmillan & Company, London, 1894, reprinted 1929), Vol. I, p. 302.

begins to be fulfilled at about point E_1, Figs. 11, 12, or about 150μ sec. after the beginning of the precursor. The time scale on Fig. 12 refers to the center of the disk, for $x_0 = 1$ ft. $= 30.5$ cm. The period of oscillation should be initially about 75μ sec., corresponding to $1/\omega = 12\mu$ sec.

These oscillations were not observed for any orientation of the disk. Near normal incidence, as in Fig. 14, No. 2, an oscillation with qualitatively the correct properties, but with too short a period (14–30μ sec.) was observed, just at the end of the oscillogram. This "postcursor" is unexplained; it may be quite unrelated to Branch B_4.

The fact that the observations fail to show these flexural vibrations should not be considered a serious objection to the theory. Their wavelength in the plate is comparable to the radius of the disk, so that here the semi-infinite plate theory is no longer applicable.

Generally speaking, the theory developed for the semi-infinite plate can be expected to fail for times appreciably after the shock front, and for long wave-length phenomena. For such cases the finite radius of the disk must be taken into account.

IX. SUMMARY AND CONCLUSIONS

The problem of the action of an underwater explosion wave on a water backed elastic plate can be divided into two parts. In the first the plate is considered as an infinite slab, with the explosion wave as an incident plane-wave transient. In the second the plate is considered as a wave guide, with the explosion wave setting the initial conditions at the edge of the plate. The first is an inhomogeneous boundary value problem, the second a homogeneous one.

The inhomogeneous case has been solved in the literature,[3] and the results of the theory are in qualitative agreement with experiment. The homogeneous problem forms the principal part of this report.

The phase-velocity curves, as functions of frequency, for the principal normal modes of a plate in water, are closely related to the corresponding curves for a plate in a vacuum. The effect of the water on the principal symmetric mode of the plate is not to change the real part of the phase velocity from its value for a plate in a vacuum, but to add a small imaginary component, or attenuation caused by radiation loss into the water. The principal antisymmetric mode is slightly attenuated at high frequencies, and strongly attenuated at low frequencies. The cut-off comes at that frequency where the phase velocity along the plate equals the velocity of sound in water.

In addition to the above two modes, which correspond to the principal modes for a plate in a vacuum, the presence of the water introduces two additional modes, one symmetric and one antisymmetric. At high frequencies their phase velocities are very close to, but a constant fraction of, the velocity of sound in water. At low frequencies the phase velocity of the symmetric mode approaches the velocity of sound in water, the phase velocity of the antisymmetric mode approaches zero. Both these modes are unattenuated; i.e., their phase velocities are all real. The antisymmetric mode accounts for most of the properties of the precursor, an oscillation which precedes the explosion wave as it travels along the plate.

Wave-Front Determination in a Unidirectional Supersonic Beam*

L. W. LABAW
Brown University, Providence, Rhode Island

(Received March 16, 1945)

An experimental method has been developed to determine the loci of equal phase in a high frequency compressional wave beam in liquids without utilizing standing wave patterns. The wave front in a supersonic beam may thus be determined simultaneously with the relative excess pressure amplitude. The electric signal generated by the piezoelectric action of a Rochelle salt microphone which has the phase of the compressional wave in the liquid is combined with a small electric signal of the same frequency having the phase of the voltage impressed across the quartz generator. The result of this superposition is an approximately sinusoidal spatial variation of the signal generated by the microphone as the latter is moved in the direction of propagation of the beam. The wave front may be traced by following a maximum of this sinusoidal variation as the microphone is moved at right angles to the direction of propagation. The experimental data, taken around 1200 kilocycles, indicates the accuracy obtainable by the method.

INTRODUCTION

COMPLETE electrostatic shielding is very difficult to obtain experimentally. Consider, for example, a well grounded steel tank $20' \times 3' \times 3'$ filled with water and located in the neighborhood of a source of electric oscillations having a frequency of approximately one megacycle. If an insulated conductor sheathed by a watertight brass tube is immersed in the water the conductor will pick up a measurable electric signal. The magnitude of this electric signal can be changed by modifying the power output of the radiofrequency generator or, for a given output, by changing the manner in which the electric generator is grounded with respect to the grounding of the steel tank.

The presence of this electric signal must be recognized in making any precise measurements in a supersonic field with a piezoelectric probe. The electric signal corresponding to the piezoelectric transformation of the excess pressure amplitude incident upon the exposed face of the microphone will combine with the electric pick-up to form the signal which is then introduced to the amplifier detector.

The resultant signal to be expected from this combination may be predicted. If the following notation is adopted,

A_1 = amplitude of the piezoelectric signal,
A_2 = amplitude of the electrical pick-up,
$\omega = 2\pi$ times the frequency,
$k_1 = 2\pi/\lambda_1$, where λ_1 is the wave-length of the supersonic signal in water,
$k_2 = 2\pi/\lambda_2$, where λ_2 is the wave-length of the electromagnetic wave in water,
X = distance measured along the direction of propagation,
t = time,
R = resultant signal, and
R_0 = amplitude of resultant signal;

then the resultant signal becomes

$$R = A_1 \exp[i(\omega t - k_1 X)] + A_2 \exp[i(\omega t - k_2 X)].$$

However, $\exp(ik_2X)$ is very nearly equal to unity for the range of X from 0 to 100 cm. Equation (1) may therefore be written approximately as

$$R = [A_1 \exp(-ik_1X) + A_2] \exp(i\omega t), \quad (2)$$

which further reduces to

$$R = R_0 \exp[i(\omega t - \alpha)],$$

where

$$R_0 = (A_1^2 + A_2^2 + 2A_1A_2 \cos k_1X)^{\frac{1}{2}},$$

and

$$\alpha = \tan^{-1}\{A_1 \sin k_1X / A_1 \cos k_1X + A_2\}. \quad (3)$$

The character of the spatial variation will be given by R_0. The maxima and minima of the oscillation will occur when $\cos k_1X$ is ± 1. At these positions, R_0 reduces to $(A_1 \pm A_2)$ or $(A_2 \pm A_1)$, depending on the relative importance of A_1 and A_2. Recognizing that A_1 will decrease with the distance away from the supersonic

* Presented in part at the New England Section of the American Physical Society meeting at Brown University on October 28, 1944.

generator, while A_2 will remain practically constant over the distance interval used, the case of $A_1 > A_2$ gives an approximately sinusoidal variation of constant amplitude whose mean value falls off as the distance, whereas the case of $A_2 > A_1$ gives a variation whose amplitude falls off as the distance but whose mean value remains practically constant.

It is apparent from the form of R_0 that the spatial oscillation will not be sinusoidal in the general case. In this connection, the value of R_0 when $\cos k_1 X$ is zero will be taken as a measure of the deviation from a sine curve. Under this condition, when $A_1 = A_2$, $R_0 = 1.41A_1$, when $A_2 = (1/2)A_1$, $R_0 = 1.12A_1$; and when $A_2 = (1/4)A_1$, $R_0 = 1.03A_1$. In all these cases a true sine curve would result in $R_0 = A_1$. If this deviation could be detected experimentally it would afford an exacting check on the above analysis.

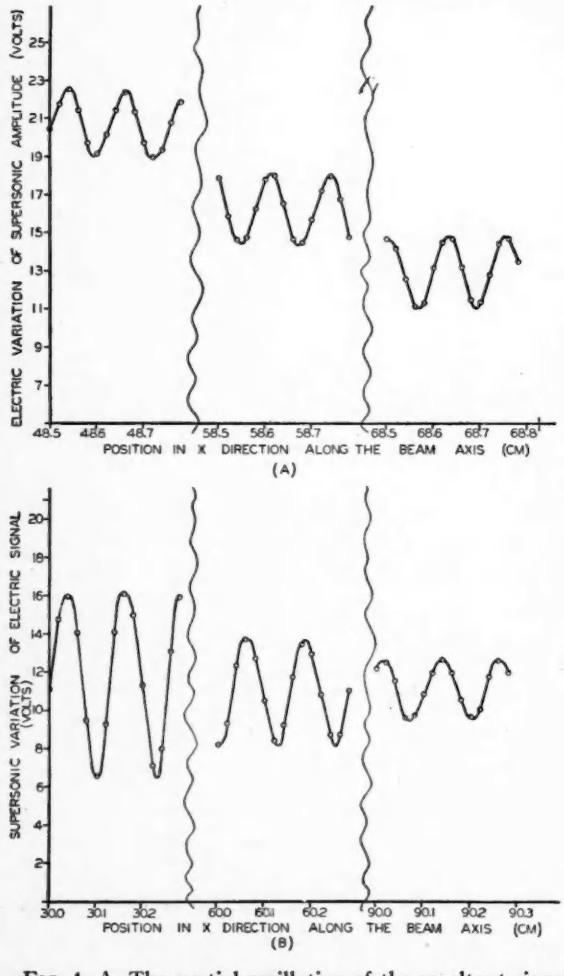


FIG. 1. A. The spatial oscillation of the resultant signal at three distances from the generator in a supersonic beam when the piezoelectric signal amplitude exceeds the electric pick-up amplitude at 1200 kilocycles. B. The spatial oscillation at three distances from the generator when the electric pick-up amplitude exceeds the piezoelectric signal amplitude at 1200 kilocycles.

MEASUREMENTS

The microphone consisted of a circular Rochelle salt plate, $\frac{1}{4}''$ in diameter, $\frac{1}{8}''$ thick, and having its thickness along the X crystallographic axis, mounted on the end of a brass tube. The front or exposed surface of the plate was covered with lead foil which was in turn soldered to the brass tube. The opposite surface of the plate was connected to the insulated conductor inside the brass tube. A length of shielded cable joined the microphone with a four-staged tuned radiofrequency amplifier, equipped with an output meter. This microphone could be moved in any of three orthogonal directions. The direction parallel to the length near one end of the $20' \times 3' \times 3'$ tank filled with water will be denoted by X, and the other two horizontally and vertically at right angles to the length will be denoted by Y and Z, respectively. An X-cut quartz crystal mounted centrally at this same end of the tank was the source of supersonic vibrations in the water at a frequency of 1200 kilocycles.

Figure 1 shows the two types of spatial variation possible. These curves were taken moving the microphone along the axis of the supersonic beam. They represent the output meter readings plotted as a function of distance. In Fig. 1B, the amplitude of the electric pick-up, A_2, is greater than that of the piezoelectric signal A_1. The readings were taken around 30, 60, and 90 cm from the quartz generator and show the predicted decrease in the amplitude of oscillation with distance together with the constant mean value. The experimental points on each curve were taken at 0.02 cm intervals. Figure 1A depicts the experimental results obtained when the piezoelectric signal A_1 was greater than the electric pick-up A_2 which was the case used in making the amplitude and phase measurements. The predicted constant amplitude of the oscillation coupled with the decrease in the mean value with distance is evident. The curves were taken at about 50, 60, and 70 cm from the supersonic source.

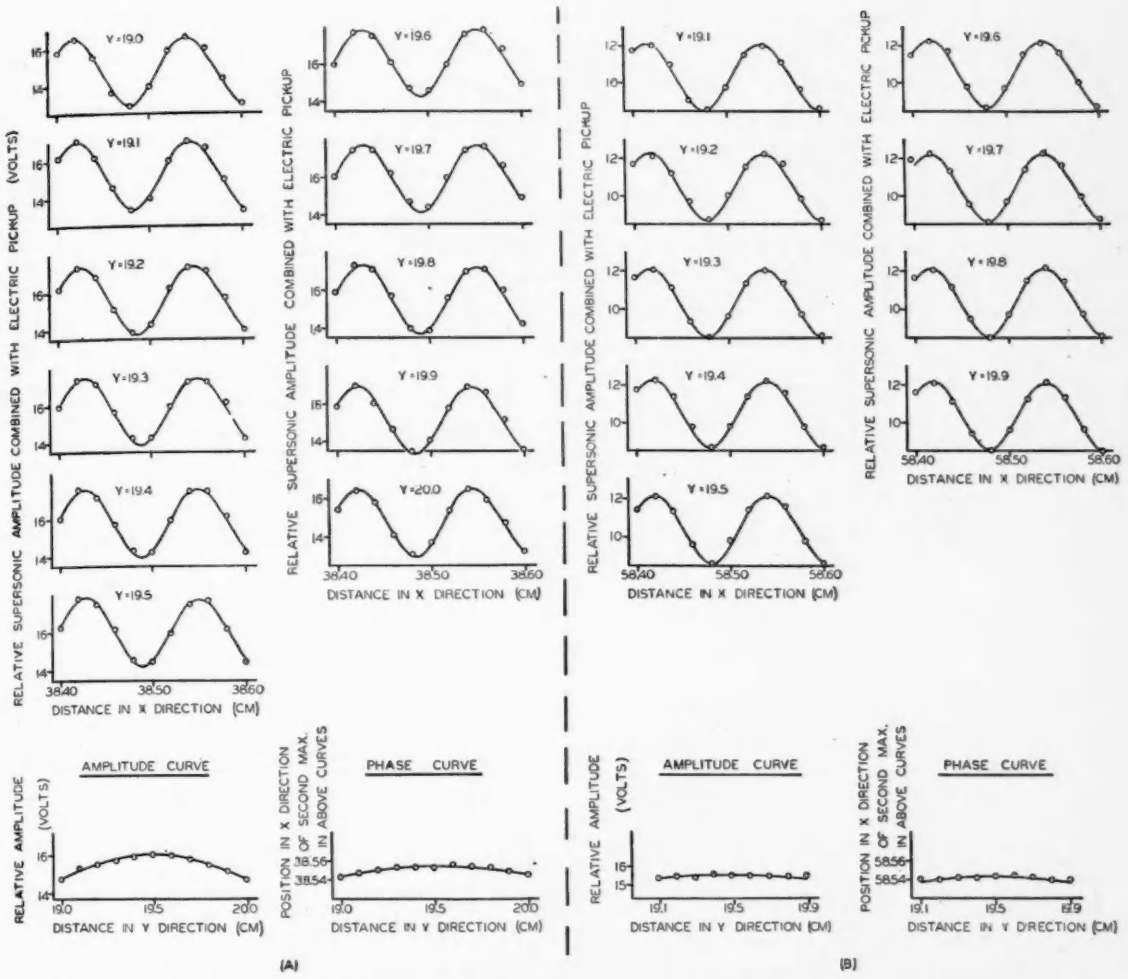


FIG. 2. A. The spatial oscillation as illustrated in Fig. 1A used to measure the relative supersonic amplitude and wave front near the beam axis about 38.5 cm from the quartz generator at a frequency of 1200 kilocycles. B. Same as A at about 58.5 cm from the generator. (The amplitude curve is drawn from the mean amplitude value at the different Y positions and the wave front, from the position of the second maximum in X for different Y positions.)

To substantiate further the correctness of the introductory analysis, the first curve in Fig. 1B where $A_1=0.43A_2$ yields $R_0=1.07A_2$ when $\cos k_1X=0$. The analysis would indicate a value of $R_0=1.08A_2$ under the same condition. An examination of the first curve reveals at a glance an increased width of the peaks compared to the troughs. The last curve in Fig. 1B together with the three in Fig. 1A seems to exhibit equal widths of peaks and troughs. This is the expected behavior since the ratio of A_1 to A_2 is so small that the inaccuracy of measurement suffices to mask the predicted change. The regularity of the variation indicates further that the supersonic beam is predominately unidirectional since any appreciable reflection would result in a distortion of this symmetry.

Measurement of the relative excess pressure amplitude, together with the wave front, is accomplished by moving the microphone in the X direction at equally spaced Y positions across the supersonic beam. The mean of the spatially fluctuating piezoelectric signal is the required relative amplitude as explained in the previous discussion, while the wave front is obtained by plotting the position along X of one particular maximum (or minimum) of the spatial oscillation as a function of the position along Y. Figures 2A and 2B illustrate this method for two different distances from the supersonic generator. In this

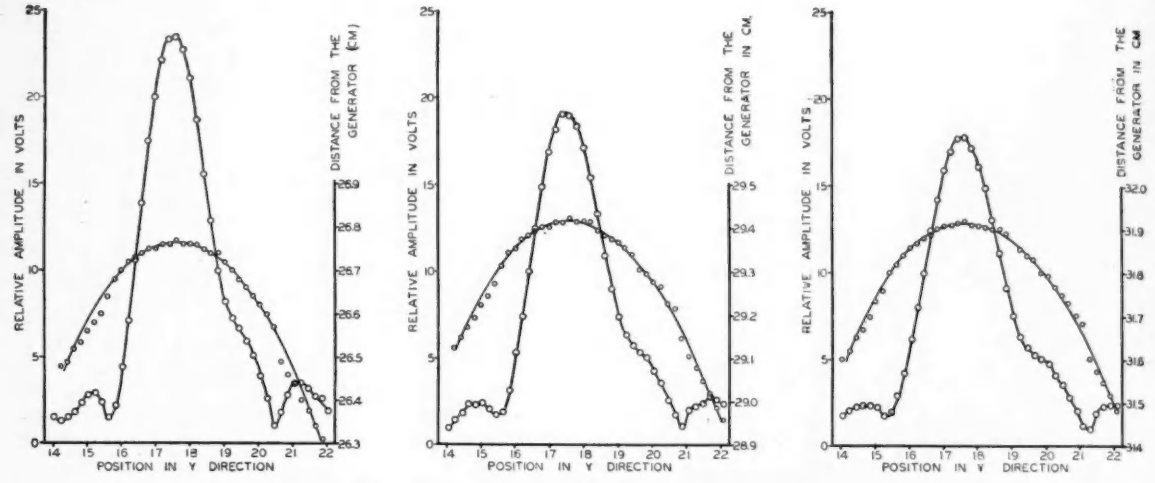

RELATIVE AMPLITUDE IN VOLTS
DISTANCE FROM THE GENERATOR (CM)
POSITION IN Y DIRECTION
RELATIVE AMPLITUDE IN VOLTS
DISTANCE FROM THE GENERATOR IN CM.
POSITION IN Y DIRECTION
RELATIVE AMPLITUDE IN VOLTS
DISTANCE FROM THE GENERATOR IN CM
POSITION IN Y DIRECTION

FIG. 3. Amplitude and equal phase curves for the principal maximum of the supersonic beam and the first secondary maxima, taken by setting on a maximum in the spatial variation visually, at three different distances from the generator as indicated by the ordinates at the left of each curve. The frequency was about 1200 kilocycles.

case the measurements were taken very close to the beam axis. The output meter readings were taken at intervals of 0.02 cm in the X direction for a fixed value of Y, and this process was repeated at 0.1 cm intervals along Y. The amplitude curve is a graph of the average of the maximum and minimum values in the variation as a function of the Y position. The X position at which the second maxima occur is used in plotting the wave front or phase curve. As is to be expected, the curvature of the amplitude trace changes considerably between Figs. 2A and 2B while the curvature of the wave front is nearly constant. Changes in the wave front of 0.01 cm are easily measurable. The measurement of smaller changes can be accomplished by taking a large number of readings, then drawing an average curve.

The method used to obtain the wave front and amplitude curves in Fig. 2 is rather tedious, as indicated by the 121 readings taken for Fig. 2A, but was dictated by the accuracy desired. A quicker method of measurement is to move the microphone along X at a given Y position while observing the output meter. One can read directly the setting of the microphone at a position of maximum signal thus obtaining the necessary data for the phase curve, provided the same maximum is followed as the microphone is moved from one Y position to another. This latter condition limits the size of the intervals along Y which are practical. This necessary interval size will change in accordance with the steepness of the wave front curve. Since the amplitude of oscillation is nearly constant, the relative excess pressure amplitude can be found by subtracting the previously determined amplitude of the electric pick-up from the output meter reading when the microphone is set at an oscillation maximum.

This method was used to obtain the data shown in Fig. 3 in which the wave front and relative supersonic amplitude are plotted for three different distances from the generator indicated by the scales to the right of the curves. The readings include the principal peak and the first secondary maximum on either side. These curves again show the expected small change in the form of the wave front for a considerable change in the amplitude curve. The curvature in the wave front is greatly magnified by the scale chosen. If the wave front were plotted on a scale comparable to that used for the position along Y it would appear practically straight. It is interesting to note that the phase curve exhibits no sharp change in curvature in passing from the principal peak to the first secondary maximum.

The method of measurement used for the data in Fig. 3 is quicker, only slightly less accurate, but requires more skill than that used for the data in Fig. 2. The choice of method is dictated

by the type of problem being attacked. The time necessary to take a given set of readings is important not only from the viewpoint of efficiency but also from that of reliability. Readings taken over too large a time interval may not be comparable because of possible changes in temperature affecting both the microphone response, the wave-length, and the circuit constant of the electronic equipment.

The author visualizes the possible use of the type of measurements outlined here as another experimental approach to problems involving diffraction, and possibly dispersion in liquids. The method is free from some of the difficulties inherent in supersonic interferometric measurements caused by beam spread and lack of piston-like source, but temperature control would admittedly be more difficult.

Observed Classical Sound Absorption in Air

W. H. PIELEMEIER
The Pennsylvania State College, State College, Pennsylvania
(Received May 26, 1945)

A brief treatment of the theory and methods of measuring absorption of sound in gases precedes the data which are presented in the form of curves. The measurements were made with a Pierce acoustic interferometer at a frequency of 1927 kc/sec. in air. This frequency is high enough to satisfy Hardy's and Krasnooshkin's required conditions to obtain reliable results by this method. The experimental result agrees with Krasnooshkin's value of $\alpha_0\lambda_0^2=(225\pm5)(10)^{-6}$ cm.

PURPOSE

THE purpose of this article is to present better acoustic interferometer data for the calculation of the absorption of sound in air.

THEORY AND DISCUSSION OF METHODS

A wide range of observed values of A in the intensity equation,

$$J=J_0\exp\left(-\frac{A}{\lambda^2}x\right),$$

for the absorption of sound in air has appeared since 1911 when Neklepajev[1] published the first experimental values in this field. Lebedev[2] who directed the work recognized the fact that only at supersonic frequencies would the classical absorption at room temperature be great enough to be measured with any degree of precision, hence such frequencies were used.

The classical (Stokes) absorption due to viscosity and heat conduction, is given by

$$m_s=2\alpha_s=A/\lambda^2=Af^2/v^2, \tag{1}$$

where

$$A=\frac{4\pi^2}{v\rho}\left[\frac{4}{3}\eta+(\gamma-1)\frac{K}{c_p}\right]. \tag{2}$$

Here m represents the *intensity* decrement, α the amplitude decrement, λ the wave-length, f the frequency, v the wave velocity, ρ the gas density, η the viscosity, γ the ratio c_p/c_v, and K the coefficient of thermal conductivity. Except for a slight dispersion, the value of A in Eq. (2) is independent of frequency and wave-length.

Neklepajev's[1] experimental value, A_e, is about twice the value given by Eq. (2). Lebedev[2] suggests as a possible explanation that the values substituted in Eq. (2) might not be applicable at these frequencies. It is more probable that the explanation should be sought in later developments such as Kneser's[3] molecular absorption and the greater apparent absorption of the complex waves from a spark.

Pumper[4] gives an expression for the *amplitude* decrement in terms of the circular frequency, ω, and of the gas pressure, p. For monatomic gases it reduces to the Stokes value,

$$\alpha_s=\frac{\omega^2}{p}\frac{1}{2v\gamma}\left[\frac{4}{3}\eta+\frac{\gamma-1}{c_p}K\right]=m_s/2. \tag{3}$$

He states that the total absorption, α_0', by a polyatomic gas should be determined by the sum of α_s and Kneser's[3] molecular absorption, α_k, i.e.,

$$\alpha_0'=\alpha_s+\alpha_k=(m_s+m_k)/2=(\mu_s+\mu_k)/2\lambda. \tag{4}$$

Schmidtmüller[5] showed that in a gas of fixed purity and temperature the experimental value of m_k at high frequencies, becomes nearly constant (independent of the frequency) and may become a small fraction of $m=m_s+m_k$. The writer's[6] best measured values of m at high frequency in air approach the value of m_s in Eq. (3), i.e.,

$$m_k\left[=M\frac{x}{1+(x^2/f^2)}\text{ of reference 6}\right]$$

becomes nearly constant while m_s increases as the square of f, the frequency.[5,6]

[1] N. Neklepajev, Ann. d. Physik **35**, 175 (1911).
[2] P. Lebedev, Ann. d. Physik **35**, 171 (1911).
[3] H. O. Kneser, J. Acous. Soc. Am. **5**, 122 (1933).
[4] E. J. Pumper, J. Phys. U.S.S.R. **1**, 411 (1939).
[5] N. Schmidtmüller, Akustische Zeits. **3**, 115 (1938); see also J. Acous. Soc. Am. **10**, 153 (1938).
[6] W. H. Pielemeier, J. Acous. Soc. Am. **16**, 273 (1945).

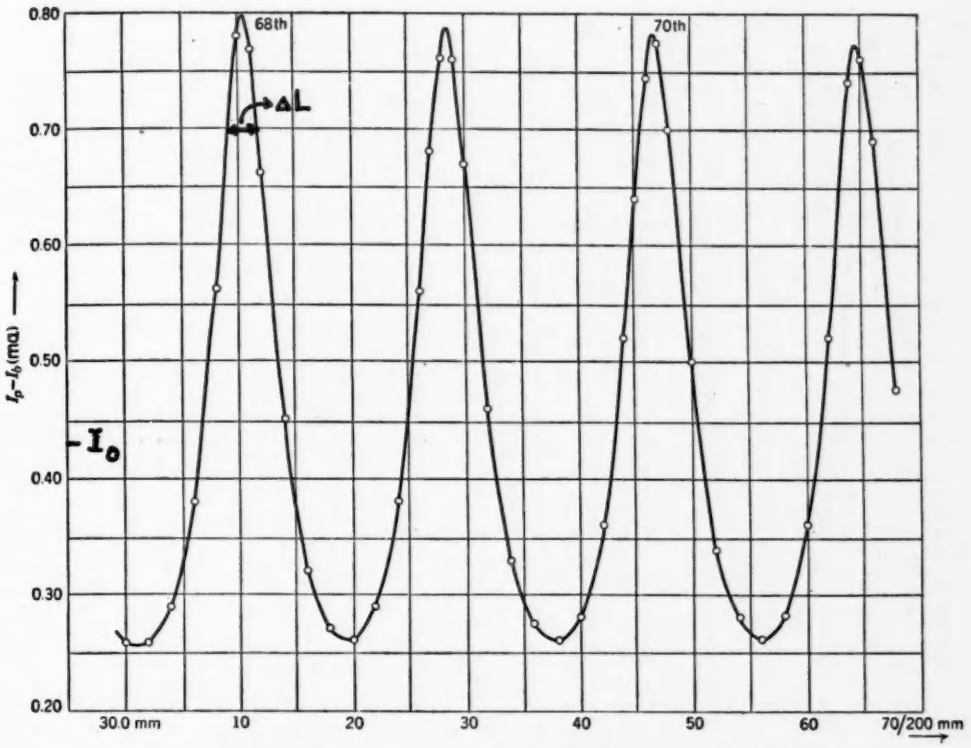


FIG. 1. (X-cut Bliley plate, $f=1927$ kc/sec.) Curve showing how the galvanometer current (plate current in the interferometer circuit minus the bias current) varies with the position of the acoustic mirror. The unbiased plate current was about 2.4 ma. Since the bias current is constant this is also the variation in the plate current. $I_0 \doteq 0.43$ ma. At 0.7 of current-peak height, (0.5 intensity or power): $\alpha_c = \frac{\Delta L}{L}\beta_c = \frac{\Delta L}{L}\, 2\pi \frac{1 \text{ cm}}{\lambda_c} \doteq \frac{\Delta L}{\lambda_c}\frac{2\pi}{L} = \frac{0.0123 \text{ mm}}{0.1800 \text{ mm}}\frac{6.28}{0.613 \text{ cm}} = 0.695 \text{ cm}^{-1} = \alpha_0$

$= \frac{\alpha_0 \lambda_0^2}{\lambda_0^2} = \frac{0.000225 \text{ cm}}{0.000324 \text{ cm}^2}$.

$\begin{cases} \alpha_0 L = (.695/\text{cm})(.613 \text{ cm}) = .427 < 1 & \text{at 68th peak} \\ \alpha_0 L = (.695/\text{cm})(1.44 \text{ cm}) = 1.00 & \text{at 160th peak.} \end{cases}$

See reference 12 for ν_0 from spectroscopic and thermodynamic data, $\lambda_0 = \nu_0/f_c$.

In the writer's 1929 publication[7] the observations made with the *radiometer* were correctly used to obtain values for m and A, but the *interferometer* data were not. They did not satisfy Hardy's[8] conditions for reliable measurements which state that the plate-current peak height with the reflector near x_1 must be less than three hundredths of the total average plate current with the reflector at x_2, also that $\mu n_2 > 5$ and that $2\mu n_1 > 5$, n being the number of half-wavelengths between the source and reflector.

In the original 1929 notes a figure like Hardy's[8] Fig. 3 was used and the published article[7] points out the existence of multiple reflections but no rigorous set of required conditions was specified. Therefore better interferometer data, that satisfy Hardy's conditions, are presented below. In the calculation of the experimental values of α_0, A, m, and $\mu(=m\lambda)$, Krasnooshkin's[9,10] acoustic interferometer theory should receive due consideration. Krasnooshkin[10] refers to the method of measuring absorption with a Pierce acoustic interferometer as the Pielemeier method. Probably it is the most accurate method known for the conditions that fit it best. Moreover the same data may also be used to calculate the velocity and to detect the presence of satellites and other disturbing features in the sound field. With very large values of m, a range of 12 mm from source to reflector is sufficient. In comparison, Neklepajev's[1] radiometer method involves greater experimental difficulties and inaccuracies. If it is to be used to measure wave-lengths a supersonic diffraction grating and a much longer acoustical

[7] W. H. Pielemeier, Phys. Rev. **34**, 1184 (1929).
[8] H. C. Hardy, J. Acous. Soc. Am. **15**, 91 (1943).
[9] P. E. Krasnooshkin, Phys. Rev. **65**, 190 (1944).
[10] P. E. Krasnooshkin, J. Phys. U.S.S.R. **7**, 80 (1943).

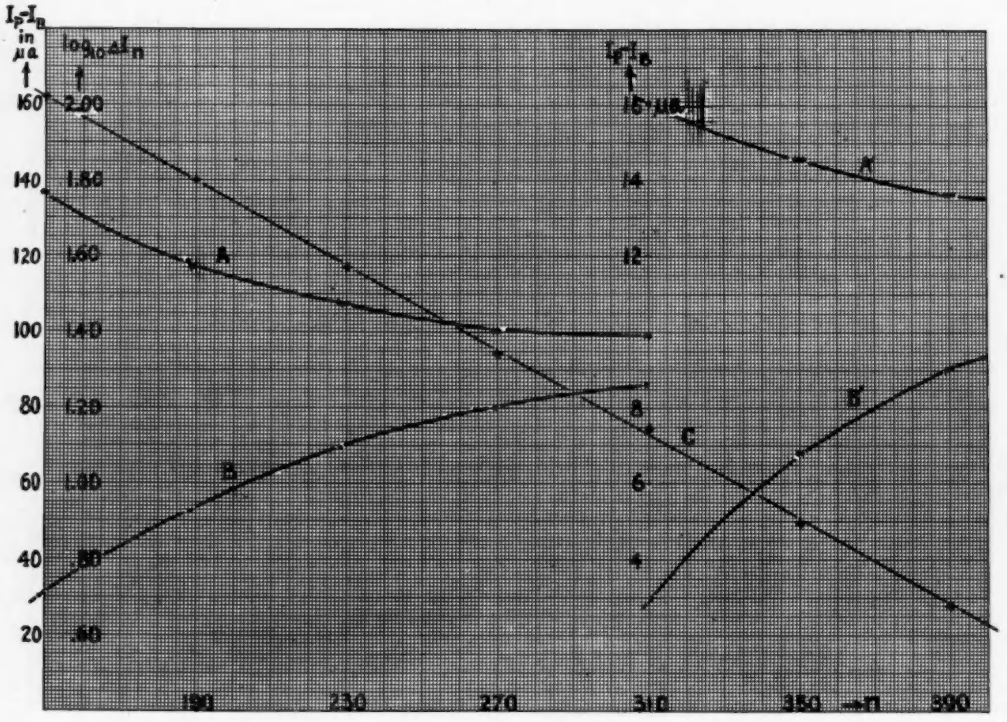


FIG. 2. Curves A and A' are drawn through the current peaks. Curves B and B' are drawn through the troughs. (See peaks and troughs in Fig. 1.) Curve C is drawn through the crosses. These show the value of $\log_{10}\Delta I_n$ where ΔI_n is the vertical separation in μa of the A and B curves at the nth peak. The sensitivity of the meter was increased 10-fold at the 310th peak. The experimental amplitude absorption per cm, α_c' is obtained from the slope of Curve C.

path is required. A long path limits its use to small values of m. For the lower frequencies Knudsen's[11] reverberation method is probably more accurate, convenient, and sensitive.

Some of the following data were taken at fairly high absolute humidity, h. This yields an experimental result which can be compared with the rather large absorption values which Knudsen[11] obtained at high humidities. The values of m_s from Eqs. (1) and (2) are very small at Knudsen's[11] 3 to 10 kc/sec. A small systematic error would loom large in comparison with these m_s-values but not with the values of $m=m_s+m_k$ when m_k is near its maximum. However, this would not explain the entire excess. At 2000 kc/sec. the expected value of m_s is about 40000 times greater than at 10 kc/sec. It should therefore, be possible to measure it more precisely at 2000 kc/sec. Lebedev[2] and others have expressed the opinion that at very high frequencies the true absorption might exceed that given by Eqs. (1) and (2) but there seems to be no reason to expect a greater excess at low than at high frequencies. If the experimental data for 1927 kc/sec. yield approximately the expected value of m, then Eqs. (1) and (2) should hold as well for lower frequencies.

Krasnooshkin's[9,10] precision method of using interferometer data is discussed below. He also gives a brief treatment[10] of another method for short paths ($\alpha_0 L<1$) which gives the approximate absorption. He uses the equation,

$$\alpha_c=(\Delta L/L)\beta_c\doteq(\Delta L/L)\beta_0, \qquad (5)$$

where ΔL = that part of the peak width due to absorption only,
L = distance from quartz to reflector,
$\beta_c=2\pi/\lambda_c=2\pi$ (the wave number) = phase change per unit length in radians/cm.

PROCEDURE, DATA, AND CALCULATIONS

Figures 1 and 2 present the experimental data. A 4-range d.c. microammeter was connected so as

[11] V. O. Knudsen, J. Acous. Soc. Am. 5, 112 (1933).

to measure I_p, the d.c. component of the plate current in the interferometer circuit. The value of I_p dropped from 3.8 to 2.2 ma when the oscillations began. A bias battery was then connected across the meter and the proper resistance so that $I_p - I_B$, the indicated time average of the current, could be reduced to nearly zero. With I_B held constant any observed changes, δI, in $(I_p - I_B)$ are really changes in I_p. As the interferometer reflector is moved away from the quartz plate which serves as sound source, δI reaches sharp *peak* values, ΔI, at the acoustic resonance positions one-half wave-length apart (See Fig. 1). For Fig. 1 $\alpha_0 L < 1$. If the peak width at $\delta I[=0.707\Delta I]$ is substituted for ΔL in Eq. (5) the value obtained for α_e is nearly 0.70 cm^{-1} which agrees with the expected true absorption, α_0. As the reflector recedes these peaks approach a sinusoidal shape. Curves A and A' in Fig. 2 are drawn through the δI peaks which are plotted as a function of the number of half-wave-lengths between the source and the reflector. Curves B and B' are drawn through the troughs. The vertical separation of the A and B curves represents the peak height, ΔI_n of the nth peak. After the 310th peak was recorded the meter sensitivity was increased tenfold. Then the remaining readings were taken. (See curves A' and B'.) Curve C represents $\log_{10}(\Delta I_n)$. Hardy's[8] conditions are satisfied beyond the 190th peak.

The experimental absorption per wave-length, μ_e' is obtained from the slope of the curve, C, by the equation

$$\mu_e' = \frac{2.303(\log_{10}\Delta I_n - \log_{10}\Delta I_{n'})}{(n'-n)/2} = 4.605\left(\frac{1.34}{240}\right) = 0.0257, \quad (6)$$

whence

$$\alpha_e' = \frac{m_e'}{2} = \frac{\mu_e'}{2\lambda} = \frac{0.0257}{2(0.01800\,\text{cm})} = 0.713\,\text{cm}^{-1}. \quad (7)$$

Krasnooshkin[9] gives a method by which he obtains the value of that part of α_e' which is only an apparent absorption, and is due to interference effects within the wave packet. He designates this part as α, and the true Stokes absorption as α_0, while Pumper[4] uses α_0 for the former. He states that $\alpha = \Delta f/8\beta_c$, where $\beta_c = 2\pi/\lambda_c$ and Δf = the range of quartz plate eigenvalues. Also Δf and f are constant for a given interferometer and quartz plate, regardless of the nature and pressure of the gas in the interferometer. The value of α is not zero unless the quartz plate vibrates as a plane piston. This is usually not the case, the amplitude usually being non-uniform over the quartz surface. However 0.713 cm^{-1} which is the above value of α_e', is equal to the sum of Krasnooshkin's value α_0, of α_s, and the value of α_k, for the above experimental conditions. $(\alpha_s + \alpha_k = 0.713\ cm^{-1})$. Thus $\alpha \doteq 0$ for this quartz plate and interferometer. The above Stokes absorption is given by

$$\alpha_s = \frac{A}{2\lambda^2} = \frac{\text{Krasnooshkin's } \alpha_0\lambda_0^2}{\lambda_0^2} = \frac{(225\pm5)(10)^{-6}\,\text{cm}}{(0.01800\,\text{cm})^2} = 0.697\,\text{cm}^{-1}. \quad (8)$$

(See Hardy, Telfair, and Pielemeier[12] for a *theoretical* value of λ_0.)

In order to compute α_k, the values of log x and of log M for 24.2°C and relative humidity, 87 percent are obtained from Kneser's nomogram.[6] Since this chart has no curve for $f = 1927$ kc/sec. the relation

$$\alpha_k = \frac{m_k}{2} \equiv \frac{m_M}{2} = \frac{M}{2}\,\frac{x}{1+(x/f)^2} = 0.0153\,\text{cm}^{-1} \quad (9)$$

is used and it is seen that $0.697 + 0.0153 = \alpha_0' \doteq \alpha_e' = 0.713$.

Krasnooshkin's[9] Eqs. (26) and (27) are presented below for convenient references.*[13]

$$\alpha_0 = \frac{\alpha_e - \Delta f/8\beta_0}{1 + f_c/2\beta_0^2}, \quad \text{(K26)}$$

$$\lambda_0 = \lambda_e \Big/ \left[1 + \frac{f_c}{2\beta_0^2}\right]. \quad \text{(K27)}$$

Here Δf = range of eigenvalues for the quartz

[12] H. C. Hardy, D. Telfair, and W. H. Pielemeier, J. Acous. Soc. Am. **13**, 226 (1942).

* Note: Δf and f_c appear as Δx and x_c in reference 10. For this treatment α_0 and α_e are primed because they include α_k and in Krasnooshkin's work they need not. For effect of $\Delta\beta$ see Michelson's visibility curves (reference 13).

[13] A. A. Michelson, *Light Waves and Their Uses*, pp. 60–83, especially p. 71.

plate $\doteq 8\alpha\beta_0$ and

$$f_c = \frac{\Delta\alpha}{\alpha} - 2\beta_0^2 = \tfrac{2}{3}\beta_0^2 \frac{\Delta(\alpha\lambda^2)}{\alpha_0\lambda_0^2}$$

$$= \text{eigenvalue corresponding to center of wave packet.}$$

CONCLUSIONS

The excellent agreement of the experimental and theoretical results shows that the value of the constant $\alpha_0\lambda_0^2 = (225 \pm 5)(10)^{-6}$ cm, as given by Krasnooshkin[10] still holds at 1927 kc/sec. This is about 30 percent greater than $A/2 = 168(10)^{-6}$ cm for 24°C as given by Eq. (2). This agreement of α_e' and α_0 together with an even better agreement of $\lambda_e = 0.01800$ cm and[12] $\lambda_0 = v_0/1927$ kc/sec. $= 0.01800$ cm favor the value $(225 \pm 5)(10)^{-6}$ cm for air at 24°C. A radiometer determination made by the writer in 1930, using a frequency of 1167 kc/sec. in air at 24.3°C gave $(220 \pm 5)(10)^{-6}$ cm.

THE JOURNAL OF THE ACOUSTICAL SOCIETY OF AMERICA VOLUME 17, NUMBER 1 JULY, 1945

A Working Standard for Sound Pressure Measurements

FRANK MASSA
Electro-Acoustic Consultant, Cleveland Heights, Ohio
(Received May 21, 1945)

This paper discusses the necessary requirements to be met in an ideal sound pressure measurement standard. It also describes the design and shows the characteristics of a new microphone that meets these requirements to a greater degree than do existing units which are generally available for use as laboratory standards. The new microphone is stiffness-controlled and has an acoustic impedance approximately equal to 0.001 cc of air throughout the entire audiofrequency range up to 20 kc. The structure approximates a rigid cylinder $\frac{3}{8}''$ diameter $\times \frac{15}{16}''$ long and is linear to sound pressure measurements up to several million dynes/cm².

INTRODUCTION

SIMPLY stated, the basic requirement of a good working standard of sound pressure measurement is that it shall give an exact quantitative reproduction of the sound pressure wave as it existed at a point before the microphone was introduced. Microphones have been designed which achieve this over-all requirement over portions of the audiofrequency range. However, the wider the frequency range over which this basic objective is realized, the better becomes the measurement standard as a useful laboratory tool.

Second in importance, after meeting the basic requirement stated above, the microphone should be simple to operate so that its use is foolproof in the hands of laboratory assistants. Simplicity of operation also means that sound pressure measurements may be made on an absolute basis with no more effort than is generally employed in obtaining only relative measurements.

In order to meet the general requirements just mentioned, several difficult conditions must be satisfied in the design of the working standard. This paper will discuss these conditions and will describe a microphone which meets them to a larger degree than previous units which have been proposed for such use.

SPECIFIC REQUIREMENTS

Several specific requirements that must be achieved in order that the working standard shall meet the ideal general requirements will be briefly discussed.

1. Flat Frequency-Response Characteristic

The convenience of having a really flat frequency-response characteristic is practically self-evident. Although variations from flatness of a microphone response are generally tolerated in the acoustic laboratory, even though considerable time may be employed in correcting for the microphone variations over the frequency range, there are certain measurements that cannot be made when such variations exist. Among such measurements are those concerned with distortion characteristics of sound generators. The apparent harmonic content would be very greatly dependent on the degree of variation from flatness of the measurement microphone.

Attempts to compensate for irregularities in a microphone response by means of electrical networks, in addition to being a generally cumbersome job, may introduce electrical phase shifts in the system that, in themselves, may cause further errors when using the microphone for the measurement of wave shape of the sound wave. An important criterion, therefore, for a good measurement standard is the extent of the frequency range over which its response is truly flat.

2. Large Dynamic Range

To be generally useful, the electrical output of a standard should be linear with respect to the sound pressure over wide ranges of magnitude. A good standard should not be restricted in its field of use only to sound pressure measurements made at large distances from intense sound sources or to the measurement of ordinary sound fields generated at a few feet from conventional radio speakers. A wide dynamic range would permit the use of the microphone in the measurement of extremely high sound pressures such as

would exist near the throat of a horn loudspeaker or inside enclosures such as sections of sound filters (mufflers) or conduits. To realize a large dynamic range, it is of primary importance that the electro-mechanical constants of the vibrating system be inherently linear for large variations in applied pressure.

3. Small Physical Size

An important criterion that must be met by a good working standard is that its physical size should be small as compared with the wave-length of the sound pressure being measured. If the structure is large enough to cause appreciable diffraction, the microphone will show sensitivity variations with angle, which will require a knowledge of the direction along which the sound wave is traveling as well as accurate alignment of the microphone with respect to the axis along which its calibration was made. Another disadvantage of a large physical size is the possibility of standing waves being established between the sound source and the microphone, which would make sound pressure measurements at high frequencies impossible.

For sound fields which are not free progressing waves with a known direction of propagation, there is no assurance of being able to measure true sound pressure unless the microphone is physically small compared with the wave-length. For example, if sound pressures are to be measured within an enclosure, such as the determination of pressure distribution along a conduit or in various compartments of an acoustic filter, it is impossible to do it accurately with a microphone of such physical size that its active area integrates a number of different pressure components acting over its entire face.

In some cases, even for work in the lower range of frequencies, it is necessary to have a microphone which is physically as small as possible, even though the wave-length is relatively large. A typical example is one in which the sound pressure measurements are to be made inside relatively small chambers in which the insertion of the microphone must not appreciably upset the physical dimensions of the enclosure.

4. Smooth Electrical Impedance

Even though a microphone may have a very flat frequency-response characteristic, it is important that its electrical-impedance characteristic be such that it easily lends itself to use with conventional electronic circuits; otherwise it might become impractical to make use of its flat open circuit sensitivity. A generally undesirable impedance characteristic is one in which large variations in magnitude occur over relatively small ranges in frequency. A very desirable type of electrical impedance for the microphone is one represented by a single circuit element over the entire audiofrequency range so that it will be easily possible to make simple adaptations of the microphone to input circuits for special uses. Some of the more common special uses include the desirability of being able easily to attenuate the microphone sensitivity uniformly over the entire frequency range at the input to the grid circuit to prevent overloading in cases where extremely high sound pressure measurements are to be made. Also, a simple circuit element will make it easy to vary the time constant of the microphone circuit, if desired. Equally simple will be the possibility of introducing filter sections at the input stage to permit special measurements in limited frequency regions when improved signal to noise ratio may be necessary in measuring very weak sound fields.

5. High Acoustic Impedance

One of the most important requirements for a sound pressure measurement standard is that its acoustic impedance shall be extremely high over the entire audiofrequency range. The importance of this particular requirement is not too generally appreciated. It is generally assumed that as long as a free field calibration of a microphone is known, it may be used as a good working standard. The fallacy in this reasoning is that the microphone only gives the correct interpretation of sound pressure when used under the identical environment in which the calibration was made. If the microphone is used under conditions other than free field (assuming free field calibration), there is no assurance that the microphone will measure true sound pressure except in the frequency region in which its acoustic impedance is high.

If the acoustic impedance of the standard microphone is not many times greater than the acoustic impedance of the environment in which

it is used, the insertion of the microphone at the point where the sound pressure measurement is to be made may greatly disturb the sound pressure which existed at the point before the microphone was introduced. Under such conditions, the measurement obtained might be in very considerable error.

DESCRIPTION OF NEW MICROPHONE

1. Mechanical Design

One of the best means of achieving the important requirement of high acoustic impedance throughout the entire audiofrequency range is to employ a stiffness-controlled vibrating system. This fact has been recognized in the design of a miniature condenser microphone[1] employing a stretched diaphragm for use as a measurement standard. However, due to the physical limitations of the strength of materials, it is impossible to stretch a diaphragm sufficiently to retain the necessary high acoustic impedance at the higher audiofrequencies.

After studying various types of stiffness-controlled transducers with a view toward eliminating the limitations of the stretched diaphragm, a mechanical system was adopted for the new standard, as indicated in Fig. 1. The active element consists of an assembly of suitably cut piezoelectric crystal plates mounted on a rigid base and surrounded by a cylindrical housing as shown in the cross-sectional view. The reason for having the outer shell separate from the base is to permit accurate sub-assembly of the crystals to the base plate with the crystals completely exposed to view. The crystal assembly is accurately located with reference to the base piece by employing suitable fixtures and when the outer shell is assembled, the crystals pass through a window opening in the housing with only small clearance between the crystals and the walls of the opening. A thin diaphragm is attached over the exposed crystal faces and the end of the housing, thus completing the assembly. It should be noted that this construction completely eliminates the cavity in front of the diaphragm, thus avoiding another possible source of high frequency distortion.

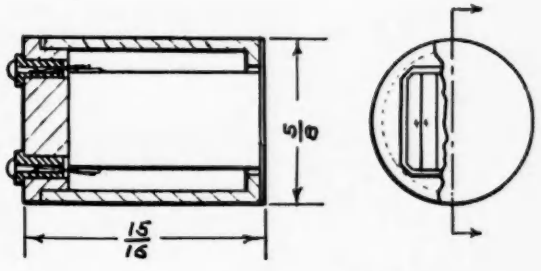


FIG. 1. Structural details of sound pressure measurement standard.

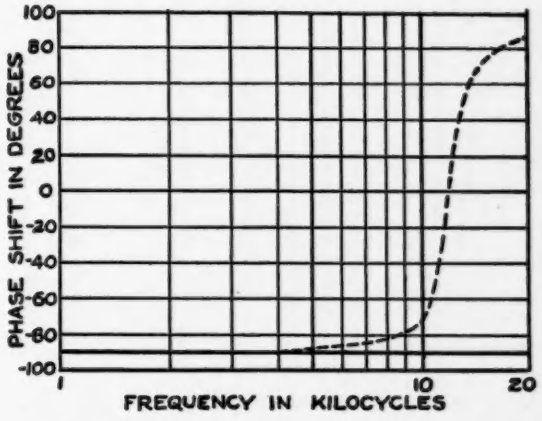


FIG. 2. Phase angle of two vibrating systems. Dotted curve—miniature highly stretched diaphragm. Solid curve—new microphone of Fig. 1.

Separate electrical connections are brought out through two contact tips, as shown, and the outer housing serves as an electrically independent enclosing shield. The reason for bringing out separate connections for the electrical leads instead of keeping one side common to ground, is to permit greater freedom in the use of the microphone in connection with a variety of input circuit conditions.

2. Acoustic Impedance of Vibrating System

The importance of having the vibrating mechanical system stiffness-controlled over the entire frequency range in which the measurement standard is to be used has already been mentioned. An indication of the amount of deviation from stiffness control of the vibrating system may be learned from the change of phase angle of the mechanical impedance of the system with frequency. For a simple vibrating mechanical

[1] W. M. Hall, J. Acous. Soc. Am. **4**, 83 (1932). Harrison and Flanders, J. Acous. Soc. Am. Supplement to July, 1932.

system, the phase angle is given by

$$\phi = \tan^{-1}\left[\frac{X_m}{R_m}\left(\frac{f}{f_r} - \frac{f_r}{f}\right)\right], \qquad (1)$$

where:

X_m = mechanical reactance of the effective mass of the vibrating system in mechanical ohms,
R_m = mechanical radiation resistance acting on the system plus any additional mechanical damping in mechanical ohms,
f = frequency of operation,
f_r = resonant frequency of the vibrating system.

A true stiffness-controlled system should show a value of ϕ from Eq. (1) equal to $-90°$ over the entire frequency range of operation. In order to compare the microphone described in this paper with a small stretched diaphragm type microphone, it will be assumed that the stretched diaphragm has an effective area of 1 sq. cm, an effective mass of 0.001 g, and a resonant frequency of 12 kc. For the new microphone, the active diaphragm area is 1 sq. cm; the effective mass of the vibrating system is approximately 2 g, and the resonant frequency is above 40 kc.

The phase shift from a pure stiffness has been computed from Eq. (1) for both of the above systems and the results are shown in Fig. 2. Mechanical damping has been neglected from the computations, which, if taken into account, would make the departure from true stiffness much greater than shown for the stretched diaphragm. The effective air load on the diaphragm

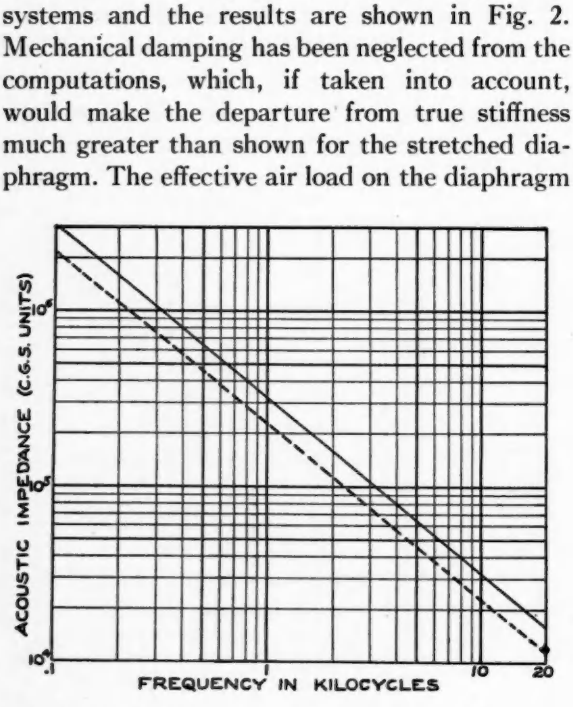


FIG. 3. Acoustic impedance of new standard microphone (solid curve). Dotted curve shows comparative impedance of 0.001 cc of air.

was found negligible and neglected; however, the decrease in the radiation resistance with decreasing frequency has been taken into account since it is to the advantage of the stretched diaphragm. The actual computations of the magnitudes of the mechanical reactances, air load, and radiation resistance have been omitted, as the interested reader may find a comprehensive quantitative treatment of these and related quantities elsewhere.[2]

The magnitude of the acoustic impedance of the new microphone is shown by the solid line in Fig. 3 and by comparison the dotted line shows the impedance of 0.001 cc of air. This very high acoustic impedance of the microphone means that true pressure measurements are assured even at very high frequencies without appreciable disturbance of the sound field. In comparison, the acoustic impedance of the miniature stretched diaphragm is only of the order of 1 percent as great, which means that difficulty may be experienced at the higher frequencies if the stretched diaphragm type is employed in high impedance environments.

3. Performance Characteristics

From the results shown in Figs. 2 and 3, it is evident that the new microphone meets the requirement of high acoustic impedance throughout the entire audiofrequency range to 20 kc. This insures true sound pressure measurements limited only by the physical size of the unit in comparison with the wave-length. The over-all size of the microphone assembly is $\frac{5}{8}''$ diameter by $\frac{15}{16}''$ long, and its physical appearance is shown in the photograph in Fig. 4. In spite of the smallness of the structure, it still cannot be considered as a true point source at frequencies in the vicinity of 10 kc.

The physical size of the microphone was arbitrarily established to give the best compromise between electrical and mechanical constants over the entire audiofrequency range. If special work is to be carried on in the region above 10 kc and a closer approach to a point source is required, the dimensions of the structure could be easily reduced to one-half those shown in Fig. 1, with an accompanying loss in sensitivity

[2] Frank Massa, *Acoustic Design Charts*, Sections 3 and 5 (Blakiston, Philadelphia, 1942).

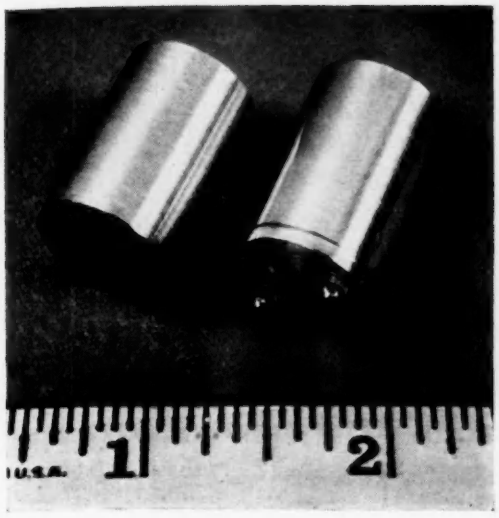


FIG. 4. Photograph of new measurement standard indicating terminal arrangements and size.

or an increase in electrical impedance. Neither of these general disadvantages would impose any serious limitation on the use of the smaller standard except at the lowest audiofrequencies, in which region the smaller structure could be replaced by the larger unit.

The maximum sound pressure that can be measured by the stretched-diaphragm type of microphone is limited by the relatively small permissible deflection of the diaphragm before non-linearity is apparent. For the diaphragm constants assumed and a nominal .002″ air gap, distortion would be evident for sound pressures of the order of 1000 dynes/cm². The new microphone is limited only by the linearity of the compliance of the crystal substance, and since the material obeys Hooke's law to pressures of the order of thousands of pounds/sq. in., there is no practical limitation to its use in measuring sound pressures of many million dynes/cm².

The electrical impedance of the microphone is essentially that of a pure condenser which means that there are no impedance irregularities over the entire audiofrequency range. Another advantage of the capacitive reactance is the possibility of employing simple frequency discriminating networks, such as shunting the grid resistor by one of lower value, if very low intensity sound pressure measurements are to be made at the mid and high audiofrequency range. For extremely high intensity sound pressure measurements which would normally produce voltages that would overload the grid circuit, it is possible to employ a shunt condenser across the microphone to decrease its sensitivity without introducing any frequency discrimination. The use of a shunt condenser will also result in holding flat response at the grid of the tube down to the sub-sonic frequency range without the use of abnormally high grid resistors.

The magnitude of the electrical capacity of the microphone as well as the absolute sensitivity may be controlled by the design of the crystal assembly and by a choice of the type of crystal employed. Structures have been designed using Rochelle salt, primary ammonium phosphate, and quartz. Rochelle salt produces maximum sensitivity and capacity, while quartz produces the lowest sensitivity and capacity. Quartz will withstand maximum temperatures of operation, whereas Rochelle salt will melt at relatively low temperatures. Another disadvantage of Rochelle salt is its very great impedance variation with temperature. The best choice of crystal for a general purpose standard that is not to be employed much above 200°F is primary ammonium phosphate. With this substance as the active element, the standard microphone has a capacity of 100 mmf and an absolute pressure sensitivity equal to 24 microvolts/dyne/cm².

The threshold of the microphone will be de-

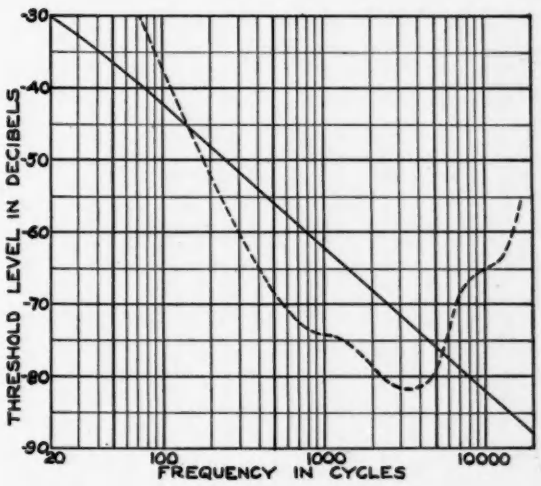


FIG. 5. Threshold level of new microphone (solid curve). Indicates sound pressure per one-cycle band width which generates a microphone voltage equal to the thermal noise of a 100 megohm grid resistor. 0 db reference level = 1 dyne/cm². Dotted curve shows the minimum threshold of the average ear after Fletcher and Munson.

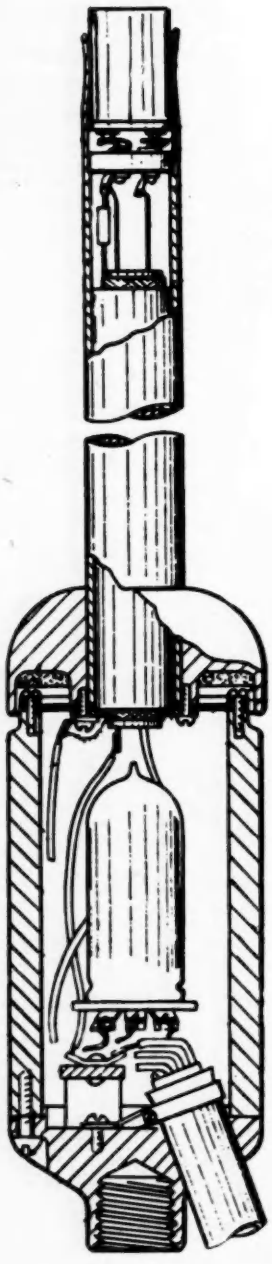


FIG. 6. Cut-away view of preamplifier design with microphone shown mounted at extremity of extension tube.

fined as the sound pressure which generates the same voltage as is produced by thermal agitation in the grid resistor. For a grid resistor of 100 megohms, a microphone employing primary ammonium phosphate crystals and having the dimensions shown in Fig. 1 has a threshold level as indicated by the solid line in Fig. 5. The dotted curve in Fig. 5 shows the minimum threshold of the average ear plotted on the same scale. (The data for the ear characteristic were obtained from Fletcher and Munson.*)

For a microphone of the same physical size employing quartz and having the same capacity of 100 mmf, the sensitivity will be approximately 15 db less than the unit described.

Because the vibrating system is designed so that the fundamental resonance occurs above 40 kc, the microphone pressure sensitivity is completely independent of frequency throughout the audio-range. In fact, due to the simple mode of vibration, the unit may be practically considered as a primary standard because the sensitivity may be derived directly from the piezoelectric constants of the crystal. Among several microphones which have been built, the measured sensitivity obtained by the reciprocity method of calibration agreed within $\frac{1}{2}$ db with the computed sensitivity derived from the piezoelectric coefficients of the crystals employed in the structure.

In Fig. 6 is shown a partially cut-away view of a pre-amplifier which has been designed for the new standard. The microphone is inserted into the open end of a $\frac{5}{8}''$ diameter tube which keeps it removed from the amplifier housing, thereby reducing diffraction errors. The base of the microphone extension tube is shock-mounted to the main portion of the amplifier housing. A vacuum tube amplifier stage is shock-mounted to the bottom base which is, in turn, threaded for mounting to a conventional microphone stand. A multi-conductor cable enters the housing, as shown, through which all the circuit connections are made at the remote end of the cable. When the amplifier assembly is removed from the microphone stand, it may be held in the hand and the microphone used as a hand probe for making sound pressure measurements.

Another application of the microphone described is as a pressure measuring element of an artificial ear. As shown in Fig. 3, the acoustic impedance of the microphone is actually greater than the impedance of .001 cc of air, which means that the unit may be inserted into small enclosures and acts essentially as a rigid wall.

* H. Fletcher and W. Munson, J. Acous. Soc. Am. **5**, 82 (1933).

Application of the Wave Theory of Room Acoustics to the Measurement of Acoustic Impedance

CYRIL M. HARRIS
Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts
(Received May 17, 1945)

A method is presented for the measurement of acoustic impedance of large areas of material, at low frequencies, under actual mounting conditions and at various angles of incidence. It is then used to check the assumption in the wave theory of room acoustics that the boundary conditions for the sound field can be expressed in terms of a normal acoustic impedance which is not a function of angle of incidence. The impedance of material covering a wall is computed from two of the room's acoustic properties, the decay constants and frequencies of the normal modes of vibration. In this way the impedance of a large area that vibrates as a panel may be measured where determinations by methods using small samples are not applicable. By using large areas, the average impedance of a number of small samples may be obtained at once. The new method has the advantage that a direct and simple check of this value of the impedance may be secured by measurements of pressure distribution of the normal modes.

INTRODUCTION

IN his paper on the wave theory of room acoustics, Morse[1] showed that it is possible to predict the acoustic properties of a rectangular room whose walls are uniformly covered, providing the absorbing properties of the wall material (and hence the boundary conditions) can be represented by a normal acoustic impedance which is not a function of angle of incidence. Since the acoustic properties of a room are not directly related to this assumption, it has been difficult to verify by direct measurements. What is needed is a complete set of data on steady-state sound distribution in a known room with walls of known impedance. Since the impedance of a material varies widely with mounting conditions, measurements of the impedance must be made under the experimental conditions which prevail when the theory is checked.

It is thus important that the assumption that normal acoustic impedance is independent of angle should be verified for materials and mounting conditions which are commonly used in practice. Beranek[2,3] measured the decay constant for a model room with material placed successively on each of three walls so that different angles of incidence were obtained. The experimental data were then compared with the decay constants calculated from impedance-tube measurements, assuming that impedance is not a function of angle. He found that the difference in mounting conditions produced a greater variation than any difference between experimental and calculated decay constants that could be associated with angle of incidence. This emphasizes the fact that to check impedance *versus* angle, measurements should be made without changing mounting conditions.

This paper presents a practical method of measurement of acoustic impedance on *large areas* of material, at *low frequencies*, *under actual mounting conditions*, and at *various angles of incidence*. This method is then used to obtain the impedance of Celotex C-4 as a function of frequency and to test for any variation of its impedance with angle of incidence.

The new method of measurement has useful engineering applications. It provides a means of evaluating acoustic impedance of material in the low frequency range under mounting conditions used in actual construction so that impedance data can be obtained for correlation with sound absorption coefficients. The latter are generally determined in reverberation rooms under mounting conditions which approximate field conditions as closely as possible. Only when both types of measurements are made under the same mounting conditions is the correlation significant.

In actual construction, large areas of material

[1] P. M. Morse, J. Acous. Soc. Am. **11**, 56 (1939).
[2] L. L. Beranek, J. Acous. Soc. Am. **12**, 3 (1940).
[3] L. L. Beranek, J. Acous. Soc. Am. **12**, 14 (1940).

that can vibrate as a whole are frequently used. In such cases the total impedance is made up of two parts: one due to the effect of penetration of air into the absorbing material, the other due to the effect of flexural vibration of the structure. Data obtained by the impedance-tube method on relatively small samples are a measure of only the former. A method of measurement of large samples which allows "panel" vibration would include both effects and would also be useful in determining the low frequency limit of impedance-tube measurements in approximating these conditions.

In measuring the impedance of absorbing material by the tube method, one finds a sample-to-sample variation in results due to differences in mounting conditions and slight differences in the samples themselves. Hence, an average of these measurements is generally taken. An additional advantage of the present method would be that this averaging process is done automatically, since areas containing a large number of samples can be used.

I. MEASUREMENT OF ACOUSTIC IMPEDANCE BY THE T-C (TRANSMISSION-CHARACTERISTIC) METHOD

If a sound source of constant output is fixed at one position in a room, then the pressure at a receiver at some other fixed point will vary as the frequency of the source is changed. When the driving frequency ν is approximately that of a normal mode of vibration ν_n, the pressure is at a maximum. A curve of pressure at the receiver *versus* frequency is called a transmission-characteristic curve. Qualitative observations of the effect of absorbing material on such a curve were noted some years ago. In 1932, Knudsen[4] described some experiments on resonance phenomena in small rooms. He observed that the introduction of absorbing material in a room produced two effects, (1) a broadening of the resonant peaks in the transmission-characteristic curve, and (2) a shift in the frequencies of the maxima (normal frequencies). Three years later, Wente[5] published transmission-characteristic curves obtained with a level recorder. He pointed out the possibility of utilizing them to predict the behavior of transient phenomena in rooms. The mathematical analysis to do this had not been carried through until the appearance of Morse's "Chapter VIII."[6] Hunt[7] then obtained the decay constants for a model room from the width of the resonance peaks in the transmission-characteristic curves.

It is possible to do more than predict transient phenomena from steady-state measurements of transmission-characteristics. One can also determine the impedance of the walls of a room. *This type of measurement of acoustic impedance will be called the T-C (transmission-characteristic) method.* The measurements are made at frequencies of the normal modes of vibration of the room, from which a curve of impedance *versus* frequency may be plotted. Suppose we consider two points on this curve close together in frequency, but for modes of vibration which strike the material at widely different angles of incidence. We would expect that if impedance varies with angle, the resulting determinations would be different, whereas if impedance is not a function of angle, they would be the same. Hence, a smooth curve of impedance *versus* frequency (containing impedance determinations for various angles of incidence) indicates that within the experimental error of the measurements, impedance is not a function of angle for the material under test. Hunt, Beranek, and Maa[8] have suggested the use of a model room with a movable wall for measuring impedance *versus* angle at constant frequency. This is a highly desirable arrangement but is practical for model rooms only and is, therefore, restricted to the higher frequency range.

Two measurements are required to determine the acoustic impedance of a material at a given frequency since it is a complex quantity. The acoustic properties of a room provide three types of measurements, any two of which could be used to determine the value of the impedance. They are the normal frequency of a mode of vibration, its decay constant, and the pressure distribution in the normal mode. We will use the first two to compute the impedance. The value

[4] V. O. Knudsen, J. Acous. Soc. Am. **4**, 20 (1932).
[5] E. C. Wente, J. Acous. Soc. Am. **7**, 123 (1935).
[6] P. M. Morse, *Vibration and Sound* (McGraw-Hill Book Company, Inc., New York, 1936).
[7] F. V. Hunt, J. Acous. Soc. Am. **10**, 216 (1939).
[8] Hunt, Beranek, and Maa, J. Acous. Soc. Am. **11**, 80 (1939).

obtained may be checked by computing the third property, the pressure distribution. If this agrees with the values of pressure experimentally obtained, the value of the impedance is correct.

From wave theory, the decay constant and normal frequency of a rectangular room can be determined for a number of cases in terms of infinite series which converge fairly rapidly (Eqs. (7)–(13), reference 1). Hunt, Beranek, and Maa[8] have given series solutions similar to Morse's. The decay constant and frequency shift produced by the introduction of material on one wall of a sound chamber is given in terms of the material's impedance providing the other walls are infinitely hard. There have been suggestions that these equations be used conversely, to obtain impedance from decay constant and frequency shift. There are some distinct disadvantages to this procedure, however, besides the fact that it would not be particularly convenient mathematically. We are interested in applying measurements of this type, not only to small-scale models, but to full-scale rooms as well, where the impedance of the other walls cannot be neglected. The necessary correction terms to the series equation complicate the solution and reduce the accuracy of the final impedance determination. Furthermore, it is necessary to measure the velocity of sound in the chamber since a difference of two frequencies is used—only one of which is measured experimentally. An error in the determination of the velocity of sound could produce a sizable error in the impedance. In order to circumvent these difficulties, another method is presented which is more accurate and easier to solve mathematically for two reasons: (a) Only one wall need be corrected for instead of five. The effect of the other four walls is eliminated. (b) The two values of frequency used in determining the impedance are both measured experimentally. If they are determined at the same temperature, a small error in the velocity of sound will produce only a small error in the determined impedance value.

In the T-C method, two sets of measurements will be made: (1) the decay constant and frequency of a normal mode when the chamber is bare, and (2) the decay constant and frequency when one wall is completely covered with absorbing material, the other walls bare. These data are then used so that the *effect of the walls normal to the material is subtracted out.* It is possible to do this since any change produced by the introduction of material on an "x"-wall, for example, produces changes in the x-variables only. If there is a door in one of the walls of the sound chamber, or some other hard non-uniformity on a wall, its effect is minimized by such a process. Only the wall opposite the material need be corrected for. Hence this procedure is decidedly advantageous from a practical standpoint.

Mathematical Analysis

In the following discussion a rectangular room will be considered whose dimensions are L_{xb}, L_{yb}, and L_{zb} in the x-, y-, and z-directions respectively. When material is placed on a wall, one of the dimensions will be altered. The subscript b will then be omitted. Only one wall will be covered with absorbing material. From the mathematical standpoint it makes no difference which one it is. Experimentally, the floor is the most convenient one to use. Our coordinate system will be chosen so that the x-axis is perpendicular to the floor ($x=L_x$) and the bare wall opposite it ($x=0$).

Some of the symbols that will be used frequently are:

p = excess pressure.
c = velocity of sound.
ρ = density of air.
λ = wave-length.
n_x = number of nodal planes perpendicular to the x-direction.
k_b = decay (attenuation) constant, floor bare.
k_c = decay (attenuation) constant, floor covered.
ν_b = frequency of a normal mode, floor bare.
ν_c = frequency of a normal mode, floor covered.
$Z=R+jX=\rho c\gamma e^{i\varphi}$ = acoustic impedance of absorbing material placed on the floor.
$Z_b=\rho c\gamma_b \exp(j\varphi_b)$ = acoustic impedance of the *bare* walls.

Assuming waves of small amplitude, the wave equation is[1]

$$(\partial^2 p/\partial x^2)+(\partial^2 p/\partial y^2)+(\partial^2 p/\partial z^2) = (1/c^2)(\partial^2 p/\partial t^2). \quad (1)$$

Substitution in (1) shows that a solution of the wave equation is

$$p=P_x P_y P_z \exp(j\omega_n t-k_n t), \quad (2)$$

where the factors are of the following form:

$$P_x = P_0 \cosh\left[(\pi x/L_x)(-\kappa_x + j\mu_x) + \psi_x\right]. \quad (3)$$

Then the frequencies of the normal modes of vibration are

$$\nu_n = \frac{\omega_n}{2\pi} = \frac{c}{2}\left[\frac{\mu_x^2-\kappa_x^2}{L_x^2} + \frac{\mu_y^2-\kappa_y^2}{L_y^2} + \frac{\mu_z^2-\kappa_z^2}{L_z^2}\right]^{\frac{1}{2}}, \quad (4)$$

and the decay constants

$$k_n = \frac{\pi^2 c^2}{\omega_n}\left[(\mu_x\kappa_x/L_x^2) + (\mu_y\kappa_y/L_y^2) + (\mu_z\kappa_z/L_z^2)\right]. \quad (5)$$

We have neglected k_n^2 in comparison with ω_n^2.

μ_x, κ_x, ψ_x = dimensionless constants whose value is determined by the material on the floor and the bare wall opposite it by the requirement that solution (3) satisfies the boundary conditions.

We will also define

μ, κ = the values of μ_x and κ_x when the bare wall at $x=0$ is infinitely hard.

μ_{xb}, κ_{xb} = the values of μ_x and κ_x when there is no material on the floor, i.e., both "x"-walls bare.

Morse[1] has shown that when one of the x-walls is hard, $2\pi\mu_x\kappa_x\lambda/L_x$ separate into two terms: δ_{x1} (in our case, depending only on the material on the floor), and δ_{x2} (depending only on the impedance of the opposite wall). Hence an alternate expression for (5) is given by

$$k_n = (c/4)\left[(1/L_x)(\delta_{x1}+\delta_{x2}) + (1/L_y)(\delta_{y1}+\delta_{y2}) + (1/L_z)(\delta_{z1}+\delta_{z2})\right], \quad (6)$$

where $(\delta_{x1}+\delta_{x2}) = 2\pi\mu_x\kappa_x\lambda/L_x$ etc.

Suppose the floor of the sound chamber is covered with material of impedance Z and the opposite wall at $x=0$ has an impedance which is infinite. Then $\psi_x = 0$ and μ_x and κ_x become μ and κ respectively. Then from Eq. (3)

$$P_x = P_0 \cosh\left[(\pi x/L_x)(-\kappa + j\mu)\right] = P_0 \cosh\,(Wx/L_x), \quad (7)$$

where $W = -\pi\kappa + j\pi\mu$.

From Newton's Second Law, the particle velocity in the x-direction at $x=L_x$ is

$$u = -(\lambda/2\pi j\rho c)(\partial p/\partial x)_{x=L_x} = -(W\lambda/2\pi j\rho c L_x)P_0 \sinh\,(Wx/L_x).$$

Applying the boundary condition that the impedance P_x/u equals Z at $x=L_x$,

$$Z = R + jX = (P_x/u)_{x=L_x} = -2\pi j\rho c(L_x/\lambda)(\coth W)/W.$$

Suppose we let $(\coth W)/W = re^{-j\beta}$ knowing κ, μ, and hence W; the values of r and β can be calculated.[9] More conveniently these values can be obtained from a conformal map or set of tables of this transformation.[10]

Hence,

$$\begin{aligned} Z/(\rho c) &= -2\pi j(L_x/\lambda)re^{-j\beta}, \\ R/(\rho c) &= -2\pi(L_x/\lambda)r\sin\beta, \\ X/(\rho c) &= -2\pi(L_x/\lambda)r\cos\beta. \end{aligned} \quad (8)$$

The impedance of the material under test is then determined from the above equations in the following way: (a) calculate μ and κ from measurements of normal frequency and decay constant, (b) using these values, determine r and β, and (c) the impedance of the material is then obtained by substitution in Eq. (8).

The values of μ and κ may be obtained algebraically from two expressions which involve physically measurable quantities, $\mu\kappa$ and $(\mu^2-\kappa^2)$.

Determination of μκ

From (5), the decay constant when the floor is bare is

$$k_b = \left(\frac{\pi^2c^2}{\omega_b}\right)\left[(\mu_{xb}\kappa_{xb}/L_{xb}^2) + (\mu_{yb}\kappa_{yb}/L_{yb}^2) + (\mu_{zb}\kappa_{zb}/L_{zb}^2)\right], \quad (9)$$

and when the floor is covered is

$$k_c = \left(\frac{\pi^2c^2}{\omega_c}\right)\left[(\mu_x\kappa_x/L_x^2) + (\mu_{yb}\kappa_{yb}/L_{yb}^2) + (\mu_{zb}\kappa_{zb}/L_{zb}^2)\right]. \quad (10)$$

Note that the subscripts b and c on a parameter

[9] The real and imaginary part of coth W may be obtained from A. E. Kennelly, *Tables of Complex Hyperbolic and Circular Functions* (Harvard University Press, Cambridge, Massachusetts, 1914).

[10] The conformal transformation $(\coth W)/W = re^{-j\beta}$ has been computed by Morse, Lowan, Feshbach, and Haurwitz. The author is indebted to them for the opportunity of using their tables prior to publication.

will refer to the value of the parameter when the floor is bare and covered with absorbing material respectively.

Subtracting (9) from (10) and making the approximation that $\omega_c=\omega_b=\omega$,

$$k_c-k_b=\left(\frac{\pi^2c^2}{\omega}\right)[(\mu_x\kappa_x/L_x^2)-(\mu_{xb}\kappa_{xb}/L_{xb}^2)]. \quad (11)$$

The term $(\mu_x\kappa_x)$ is due to the absorbing material and the opposite wall. Since the latter is hard, the quantity $(\mu_x\kappa_x)$ is separable into two terms

$$(\mu_x\kappa_x)=\mu\kappa+(\mu_{xb}\kappa_{xb})/2. \quad (12)$$

In Eq. (6) if we assume that all the bare walls in the chamber are similar and hard, all the δ's (with subscripts) are equal and will be replaced by δ. Equation (6) then becomes

$$k_b=c\delta(\sigma_xL_{yb}L_{zb}+\sigma_yL_{zb}L_{xb}+\sigma_zL_{xb}L_{yb})/(2L_{xb}L_{yb}L_{zb}), \quad (13)$$

where $\sigma_x=\frac{1}{2}$ for $n_x=0$
$\sigma_x=1$ $n_x>0$, etc.

Then

$$\frac{\pi^2c^2}{\omega}(\mu_{xb}\kappa_{xb}/L_{xb}^2)=\frac{c\sigma_x\delta}{2L_{xb}}. \quad (14)$$

Hence, combining (11), (12), (13), and (14)

$$\mu\kappa=\frac{\omega L_x^2}{\pi^2c^2}\left[(k_c-k_b)+\frac{\sigma_xk_bL_{yb}L_{zb}}{2(\sigma_xL_{yb}L_{zb}+\sigma_yL_{zb}L_{xb}+\sigma_zL_{xb}L_{yb})}\right]. \quad (15)$$

Determination of $\mu^2-\kappa^2$

The value of $\mu^2-\kappa^2$ is a function of the change in frequency produced by the introduction of material in the sound chamber. It may be obtained as follows. From Eq. (4), when the chamber is bare

$$(2\nu_b/c)^2=\frac{\mu_{xb}^2-\kappa_{xb}^2}{L_{xb}^2}+\frac{\mu_{yb}^2-\kappa_{yb}^2}{L_{yb}^2}+\frac{\mu_{zb}^2-\kappa_{zb}^2}{L_{zb}^2},$$

and when the floor is covered

$$(2\nu_c/c)^2=\frac{\mu_x^2-\kappa_x^2}{L_x^2}+\frac{\mu_{yb}^2-\kappa_{yb}^2}{L_{yb}^2}+\frac{\mu_{zb}^2-\kappa_{zb}^2}{L_{zb}^2}.$$

Hence by subtraction

$$4(\nu_b^2-\nu_c^2)/c^2=\left[\frac{\mu_{xb}^2-\kappa_{xb}^2}{L_{xb}^2}\right]-\left[\frac{\mu_x^2-\kappa_x^2}{L_x^2}\right]. \quad (16)$$

We wish to determine $\mu^2-\kappa^2$, the value that would be obtained if the wall opposite the material were infinitely hard. To do this, (16) is expressed as functions of $\mu^2-\kappa^2$ and the bare wall impedance $\gamma_b\exp(j\varphi_b)$.

If the impedance of the acoustical material on the floor $\gamma e^{j\varphi}$ is large, as it is in the low frequency range, we may express the terms on the right-hand side of Eq. (16) as series expansions (reference 1, Eqs. (7) and (8)). Then

$$\begin{aligned}(\mu_{xb}^2-\kappa_{xb}^2)&=n_x^2+\frac{8L_{xb}\sigma_x}{\pi\lambda}\left(\frac{\sin\varphi_b}{\gamma_b}\right),\\ (\mu_x^2-\kappa_x^2)&=(\mu^2-\kappa^2)+\frac{4L_x\sigma_x}{\pi\lambda}\left(\frac{\sin\varphi_b}{\gamma_b}\right).\end{aligned} \quad (17)$$

Thus, solving (16) and (17) for $(\mu^2-\kappa^2)$

$$\frac{(\mu^2-\kappa^2)}{L_x^2}=\frac{n_x^2}{L_{xb}^2}-\frac{4(\nu_b^2-\nu_c^2)}{c^2}+\frac{8\sigma_x}{\pi\lambda L_{xb}}\left(\frac{\sin\varphi_b}{\gamma_b}\right)-\frac{4\sigma_x}{\pi\lambda L_x}\left(\frac{\sin\varphi_b}{\gamma_b}\right);$$

and when $\frac{\sin\varphi_b}{\gamma_b}\ll\frac{\sin\varphi}{\gamma}$,

$$\frac{(\mu^2-\kappa^2)}{L_x^2}=\frac{n_x^2}{L_{xb}^2}-\frac{4(\nu_b^2-\nu_c^2)}{c^2}. \quad (18)$$

Measurement of Impedance of the Bare Walls

The value of the impedance of a material placed on the floor of a sound chamber has been expressed in terms of the frequencies of the normal modes of vibration of the chamber and the damping constants associated with these modes by Eqs. (15) and (18). Each equation contains a correction term due to the wall opposite the material on the floor. (The effect of the other bare walls is completely eliminated.) In the first of these equations the correction term can be easily computed. In the second, it can be neglected in general if the walls are sufficiently hard. Otherwise it can be computed

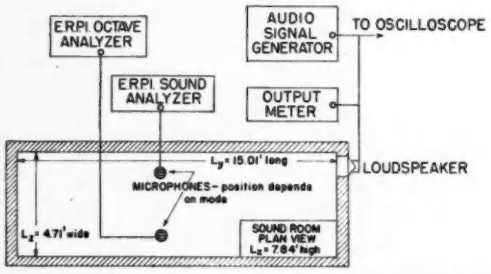


FIG. 1. Block diagram of experimental apparatus.

from a knowledge of the bare wall impedance which may be measured in the following way.

As stated previously, the complex impedance of a material at a given frequency may be determined from two of three acoustic properties of a sound chamber: the frequency of a normal mode of vibration ν_n, its decay constant k_n, or the pressure distribution of the mode. When sound absorbing material is added to a chamber, the frequency and decay constant are the most convenient quantities to use. Here, however, we are not adding material to the chamber, but wish to determine the impedance of the bare walls. To make this computation, using ν_n and k_n would necessitate a comparison of actual measurements with values calculated for the same room with infinitely hard walls. The velocity of sound would have to be measured accurately since the difference in frequency between these two cases is extremely small, and large errors in the impedance determination of the bare walls might be obtained. Consequently, here it is more accurate to use the decay constant and pressure distribution of a mode. The first of these quantities k_b provides one relationship between γ_b and φ_b. A second relationship is provided by the ratio of the magnitude of the pressure at a minimum $|P_{\min}|$ to the magnitude of the pressure at $x=0$.

If all walls of the chamber are hard and have the same acoustic impedance $Z_b=\rho c\gamma_b \exp(j\varphi_b)$, then from reference 1, Eq. (8), when $n_x\neq 0$, the damping constant δ is

$$\delta=\frac{4\cos\varphi_b}{\gamma_b}.$$

Combining this with (13), we obtain one relation between γ_b and φ_b.

$$\frac{k_b V_b}{2c(A_x+A_y+A_z)}=\frac{\cos\varphi_b}{\gamma_b}, \tag{19}$$

where $V_b=L_{xb}L_{yb}L_{zb}$, and $A_x=\sigma_x L_{xb}L_{yb}$, etc. We obtain the second relationship from the pressure distribution in the x-direction. Since we have symmetrical boundaries for the bare room, it is most convenient to choose our coordinate system with its origin at the center of the room and walls at $x=\pm L_{xb}/2$, etc. Substitution in the wave equation shows that the following are solutions satisfying the boundary conditions

$$P_x=P_0\sinh\left[(\pi x/L_{xb})(-\kappa_{xb}+j\mu_{xb})\right] \qquad n_x=\text{odd integer},$$

$$P_x=P_0\cosh\left[(\pi x/L_{xb})(-\kappa_{xb}+j\mu_{xb})\right] \qquad n_x=\text{even integer},$$

where n_x is the number of nodal planes perpendicular to the x-axis. (It is interesting to note that in a room whose walls all have the same impedance, the pressure at the center of the room is zero for n_x odd, irrespective of the value of the impedance.)

Suppose we consider the case where n_x is an even integer. The ratio of the magnitude of the pressure at a minimum to the magnitude at $x=0$ is given by

$$|P_{\min}|/|P_{x=0}|=\cosh\left[(\pi x/L_{xb})(-\kappa_{xb}+j\mu_{xb})\right], \tag{20}$$

where $(-\kappa_{xb}+j\mu_{xb})$ has been evaluated in terms of $\gamma_b\exp(j\varphi_b)$ by Eq. (8), reference 1, for $n_x>0$ and Z_b large

$$(-\kappa_{xb}+j\mu_{xb})=-\frac{4L_{xb}\cos\varphi_b}{\lambda\pi n_x\gamma_b}+j\left[n_x+\frac{4L_{xb}\sin\varphi_b}{\lambda\pi n_x\gamma_b}\right]. \tag{21}$$

Equations (19)–(21) then uniquely determine the boundary impedance from the measurement of a peak-width and a pressure ratio.

II. EXPERIMENTAL PROCEDURE

A block diagram of the experimental equipment is shown in Fig. 1. Either of two audio-

signal generators is used, a Beat-Frequency Oscillator provided with an incremental-pitch condenser for measurements of ν_n and k_n, or an R-C Type Oscillator for pressure-distribution measurements. Both are powered from voltage stabilized supplies. A 12″ loudspeaker is mounted flush in one corner of a small room whose plan view is shown in Fig. 1. When material is introduced in the chamber it completely covers the floor. Celotex C-4 tiles were selected for test and were mounted on $\frac{3}{4}$″ Plywood panels approximately 3 feet by 4 feet, with Atlas Cement.

Two W.E. 633A Microphones measure sound pressure-levels in the room. One is connected to an E.R.P.I. Sound Analyzer provided with band-widths of 5, 20, and 200 c.p.s. An E.R.P.I. Octave Analyzer measures the output of the other. These microphones are very much smaller than the wave-lengths involved in the acoustic measurements. It is essential that an analyzer be used as a level indicator to eliminate the effects of loudspeaker distortion which may otherwise provide a considerable source of error at low frequencies. The analyzer band-width is chosen so that its frequency-response characteristic is flat over the range used in measuring the decay constants (less than a few c.p.s.).

Microphone placement in the sound chamber plays a very important role in determining the accuracy with which the frequency, decay constant, and pressure distribution of a normal mode can be measured. Data can only be taken on isolated modes of vibration. A few will be well separated from others because of the geometry of the room. Some can be effectively isolated by proper placement of the microphone and/or the source. This can be seen from Eq. (34.7), reference 6 which gives the pressure at some point (x, y, z) if the loudspeaker is at (x_0, y_0, z_0)

$$p = Ae^{j\omega t} \sum_n \frac{(\omega/\Lambda_n)\psi_n(x_0, y_0, z_0)}{(\omega_n k_n) + j(\omega^2 - \omega_n^2)} \psi_n(x, y, z) \quad (22)$$

for $k_n^2 \ll \omega^2$, where A is a constant and ψ_n, a set of characteristic functions. One should place the microphone so that it is at a pressure maximum for the mode under consideration and at a minimum for the neighboring ones.

If we consider an isolated mode of vibration, the summation sign in (22) is removed. Then if the driving frequency is varied, there will be a peak at ν_n in the response curve which is practically symmetrical. Under these conditions the resonant frequency is independent of microphone position. However, if there are contributions from neighboring modes, (22) indicates that the resonance curve may be very non-symmetrical and *the frequency ν_n will apparently be a function of position*. Consequently, a plot of the response curve may indicate whether or not a mode is isolated. A more rapid and more sensitive method of detecting contributions from neighboring modes is as follows. The frequency of the peak response is measured with the microphone at the absorbing material and then at the bare wall opposite it. If the frequency is not the same in both cases, then the mode is not isolated and cannot be used for impedance measurements.

Measurement of Decay Constant k_n

The decay constant of a normal mode of vibration can be determined from the width of the resonance peak of the mode. From (22) the pressure has a maximum value at the angular frequency ω_n (providing $k^2 \ll \omega^2$). Suppose we let ω'' and ω' be frequencies on either side of the resonance peak where the pressure amplitude has the value of $|P_{\max}|/2^{\frac{1}{2}}$. It follows immediately that the decay constant can be expressed in terms of these angular frequencies[7]

$$k_n = (\omega'' - \omega')/2.$$

All decay constants were measured in this way. The values of ω'' and ω' were obtained by shifting the frequency of a beat-frequency oscillator from resonance to a point where the pressure was reduced by a factor of $2^{\frac{1}{2}}$. The difference between these frequencies was then measured with a calibrated incremental-pitch condenser, a device for varying the frequency of an oscillator by known amounts. The condenser was calibrated in terms of frequency differences over a small range by comparison with standard frequencies (discussed in the next section). The peak-widths measured were of the order of one or two cycles, and the incremental-pitch condenser could be read to 0.04 c.p.s. In making these measurements, the stability of the oscillator is important. From this standpoint it was found desirable to substitute batteries for the filament transformer

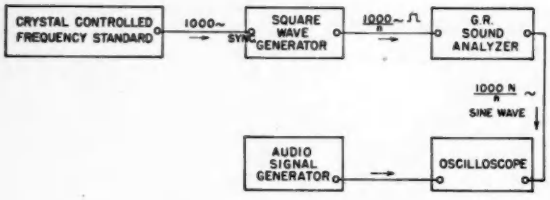


FIG. 2. Block diagram of frequency measurement equipment.

in the commercially made beat-frequency oscillator used. A voltage-regulated power supply for the oscillator is essential.

An independent check on the ratio of the values of k_b and k_c may be obtained as follows. Equation (22) shows that for a single mode of vibration and a fixed position of the source and microphone, the pressure amplitude at resonance is

$$P=(\text{constant})/k_n.$$

And hence for small values of absorption

$$\frac{P_b}{P_c}=\frac{k_c}{k_b}=\frac{(\omega''-\omega')_c}{(\omega''-\omega')_b}, \tag{23}$$

where the subscripts "b" and "c" refer to the quantities measured with the room bare and with the floor covered, respectively. As an example, for the (1,2,0) mode, the value of P_c/P_b was found to be 1.38. The ratio of the peak-width when the material was in the room to the value when it was removed was 1.41, within the experimental error of the preceding ratio.

Equation (23) can also be used to measure the peak-width of a bare chamber with exceedingly hard walls. In this case the peak-width is very small and therefore difficult to measure accurately. However, it is easy to measure the pressure levels at a fixed point with and without the floor covered with material. Since the peak-width is relatively wide when the floor is covered, (23) gives the peak-width of the chamber when bare with better accuracy than could be obtained by direct measurement.

Measurement of Frequency

In determining acoustic impedance by the T-C method, the normal frequencies of the room, ν_b and ν_c (with the floor bare and covered, respectively), must be measured with considerable accuracy. Both must be determined at the same temperature to avoid any shift in frequency of a normal mode due to a change in the velocity of sound. Consequently, they should be evaluated within a short time of each other.[11] As in the measurement of decay constants, proper microphone placement greatly increases the accuracy of the results.

An unknown frequency was evaluated by measuring the difference between it and a standard frequency very close to it. This was done with the incremental-pitch condenser described in the preceding section. For precise results this difference should be kept small. If it were zero, then the unknown frequency would be measured to the accuracy of the standard. This means that for good results, there should be available (a) a method of comparing two frequencies, and (b) a large number of frequency standards so that there will always be one close to the unknown.

All frequency comparison was done by means of Lissajous figures on a cathode-ray oscilloscope. A large number of frequency standards was obtained from a single 1000 c.p.s. G.R. Crystal Controlled Frequency Standard by an arrangement shown in Fig. 2. The one-kilocycle signal is fed into the synchronizer of a square-wave generator which can be set to operate at any nth submultiple of 1000 c.p.s. Thus a square wave, rich in harmonics, of $1000/n$ c.p.s. is obtained. The Nth harmonic is obtained by means of a G.R. Sound Analyzer. Tuning the (harmonic) analyzer to $1000N/n$ c.p.s. allows only this frequency to pass. The output of the analyzer then provides the oscilloscope with a sine-wave of this frequency so that it may be compared with the unknown frequency.

III. RESULTS OF IMPEDANCE MEASUREMENTS BY THE T-C METHOD

The acoustic impedance of Celotex C-4 was measured at frequencies corresponding to five

[11] When this experimental work was first started, a device was used to measure any shift in the velocity of sound that might occur. It consisted of a metal tube with a W.E. 555 Speaker Unit on one end, sealed by a microphone on the other. The driving frequency of the sound source was set to the resonant frequency of the column. Any change in the velocity of sound would produce a shift in the resonant frequency which could be easily detected. This apparatus was found to be unnecessary when measurements of ν_b and ν_c are made in succession.

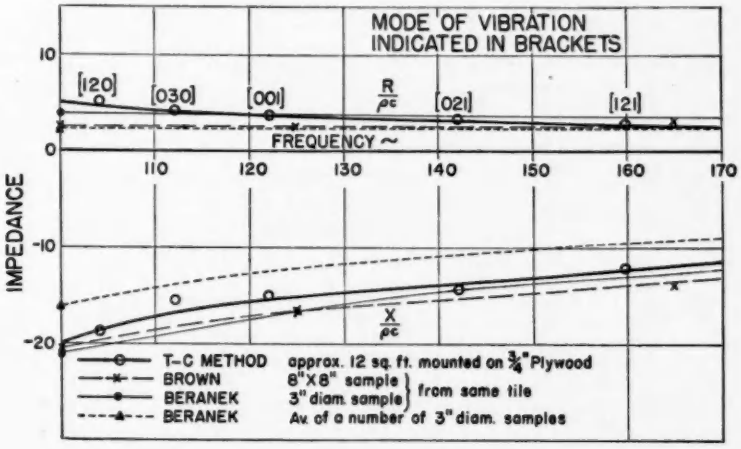


FIG. 3. Acoustic impedance of Celotex C-4 *versus* frequency. Each experimental point obtained by the T-C method gives the value of the impedance at a frequency and angle of incidence corresponding to a normal mode of vibration indicated above the point. The angle of incidence for the mode may be found in Table I. Impedance data given by Beranek and Brown were obtained for small samples by the tube method and are for normal incidence.

normal modes of vibration of the room in which the material was placed. When material was introduced in the room, the normal frequencies ν_b shifted by an amount $\Delta\nu$ and the damping constant increased by an amount (k_c-k_b). Experimental values of these quantities are given in Table I.

Results of measurements on Celotex C-4 by the T-C method have been plotted in Fig. 3. Data have been taken by other observers[3,12] on small samples of this material by the impedance-tube method. One would not expect these measurements to give the same results as those obtained by the T-C method because of the extreme difference in size of the samples and mounting conditions. For purposes of comparison the impedances obtained by the tube method have also been plotted in Fig. 3.

Having determined the impedance of the material covering the floor of the test room from μ and κ (Eq. (8)), the angles of incidence can be computed from[1]

$$\theta=\cos^{-1}\left[\lambda(\mu_x-\kappa_x)^{\frac{1}{2}}/2L_x\right].$$

Values of θ for the modes of vibration are given in Table I. It is interesting to note that the so-called "grazing" modes (0,3,0), (0,0,1) and (0,2,1) differ from being truly grazing ($\theta=90°$) by as much as 6°. Even though the angles of incidence vary considerably for the various modes, the calculated values of the impedance form a fairly smooth curve, Fig. 3. Since the deviations from this curve do not appear to be associated with the angle of incidence, and since points close together in frequency but widely differing in angle give nearly the same value of impedance, one can conclude that *the assumption* that the boundary impedance can be represented in terms of a normal acoustic impedance which is not a function of angle *is valid* for the material under test within experimental error.

[12] R. L. Brown, Doctorate Thesis, M.I.T. (1942).

IV. PRESSURE DISTRIBUTION MEASUREMENTS

Experimental measurements of pressure distribution in a normal mode of vibration can be used to check the values of impedance obtained by the T-C method. From (3), the distribution

TABLE I. Measurement of acoustic impedance by the T-C method.*

Celotex C-4 mounted on ¾" Plywood panels						
Mode	ν_b c.p.s.	$\Delta\nu$ c.p.s.	$\frac{k_c-k_b}{\text{sec.}}\cos^{-1}\frac{(\mu_x^2-\kappa_x^2)^{\frac{1}{2}}\lambda}{2L_x}$		$\frac{R}{\rho c}$	$\frac{X}{\rho c}$
1 2 0	104.1	−0.47	1.38	46°	5.1	−18.6
0 3 0	111.8	−0.75	1.02	87°	4.1	−15.3
0 0 1	122.1	−0.79	1.48	86°	3.6	−15.0
0 2 1	141.8	−1.07	2.86	84°	3.2	−14.5
1 2 1	159.5	−1.39	3.60	64°	2.9	−12.1

* The first integer in the normal mode indicates the number of pressure nodal-planes parallel to the wall covered with absorbing material.

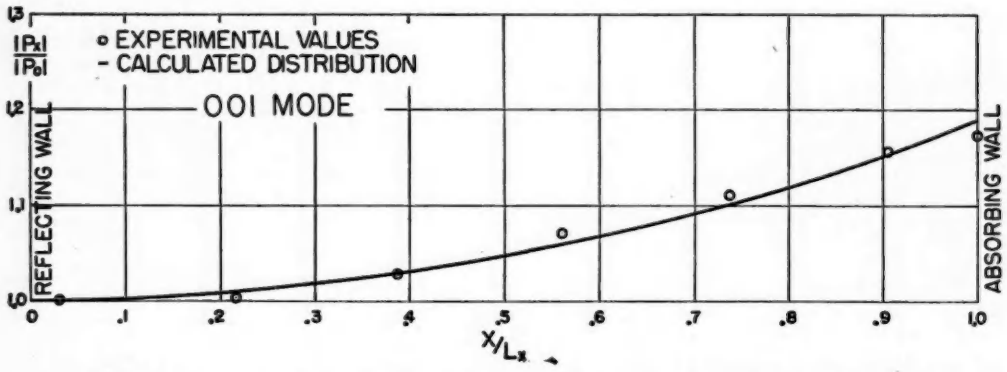


FIG. 4. Relative-pressure amplitude *versus* position between two parallel walls for a mode having no pressure nodes perpendicular to the x-direction.

in the x-direction is calculated using the measured values of impedance of the material Z and of the bare wall Z_b. If these values are correct, the calculated distribution will agree with the experimental pressure measurements. Comparisons for two modes of vibration are given in Figs. 4 and 5. The apparatus for determining the experimental points on these curves is shown in Fig. 1. An E.R.P.I. Sound Analyzer with a 5 c.p.s. band-width was used as a pressure-level indicator, thus eliminating the effect of loudspeaker harmonics. Here as in previous measurements the position of the microphone is important since the mode of vibration, whose distribution is being measured, must be well isolated if this distribution is to be significant. Throughout the measurements the frequency of the audio-signal generator was carefully maintained at the resonant frequency of the normal mode.

The solid curve in Fig. 4 is the calculated pressure distribution for the (0,0,1) mode. It is representative of the distribution obtained for "grazing" modes when one wall is covered with an impedance which has a large negative reactance. The effect of the material is to increase the pressure at the wall as shown. Experimental points indicate good agreement with the calculated values. This serves not only as a check on the value of the impedance but as an additional verification of the wave theory of room acoustics.[1]

A typical pressure-distribution curve for a mode with one nodal plane parallel to a wall covered with material that has a large negative reactance is shown in Fig. 5, the (1,2,0) mode. The effect of absorbing material of this kind is to pull the nodal-plane toward it.

CONCLUSIONS

By the use of the T-C method, it has been possible to measure the acoustic impedance under actual mounting conditions of Celotex C-4 which completely covered the floor of a test room. Values of impedance obtained by this method have been used to check the normal acoustic-impedance assumption in the wave theory of room acoustics, at low frequencies. Results indicate that the acoustic impedance for this material is not a function of angle within the limits of experimental error.

Measurements are made at frequencies corresponding to normal modes of vibration of the test room. It is not necessary that the modes be "well spaced," instead a few must be well isolated. This fact does not restrict the T-C method to any size room, but rather the size is dependent on the frequency range to be investigated. At low frequencies, for example, a reverberation room could be used. Between 100 and 200 c.p.s. most absorbing materials have large impedances and therefore the decay constant for the chamber is small. Hence, a mode having no neighbors within 5 or 10 c.p.s. could be used. At higher frequencies a small chamber or model room would be useful in isolating the modes. Since the normal frequency of a mode must be measured accurately, a standard of frequency is required.

With the T-C method of acoustic impedance measurement it is possible to use large samples

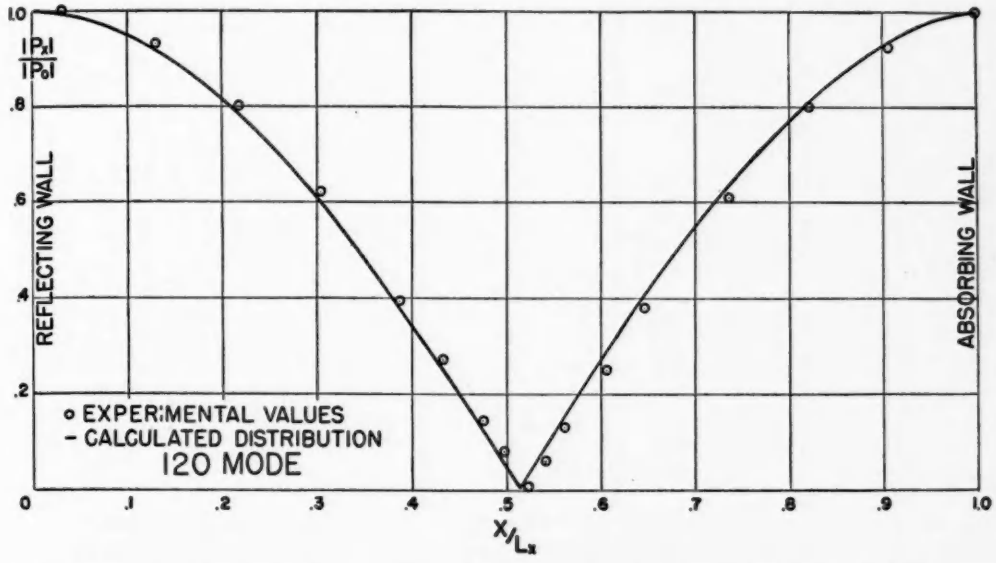


FIG. 5. Relative-pressure amplitude *versus* position between two parallel walls for a mode having one pressure node perpendicular to the *x*-direction.

duplicating standard types of mounting conditions used in actual construction and in reverberation measurements. One could, therefore, obtain a correlation between absorption coefficients and impedance in the low frequency range under the same mounting conditions. This method would also be useful for making measurements on wall material when the surface vibrates as a whole. In this case impedance-tube measurements are not significant in representing the total impedance of the material since they are made on relatively small samples which allow little or no flexural vibration. The measurements on larger samples would be useful in indicating the lower frequency limit of impedance-tube data in approximating these measurements.

The value of impedance obtained by the T-C method may be easily and directly checked by pressure-distribution measurements.

ACKNOWLEDGMENTS

The author wishes to express his appreciation and thanks to Professors P. M. Morse and H. Feshbach for their encouragement, guidance, and for many valuable suggestions during the course of this research work. He is also grateful to Professor R. D. Fay and Mr. M. Lax for many helpful and stimulating discussions.

A Frequency Standard for Use at High and Low Frequencies

ROGELIO P. MCLOUGHLIN*
Translated from the Spanish by NEIL E. HANDEL
Underwater Sound Laboratory, Harvard University, Cambridge, Massachusetts

A secondary frequency standard is described for use at high and low frequencies, consisting of a tuning-fork oscillator which controls an Abraham and Block multivibrator. An amplifier of adequate selectivity allows the selection of any harmonic of the fundamental voltage. This is accomplished by the use of a multivibrator and a regenerative amplifier of high selectivity which allows selection of any harmonic of the fundamental frequency in the band 1 kc to 2 mc. The tuning-fork oscillator presents the following advantages: 1. Possibility of adjusting the frequency by varying the amplitude of oscillation of the tuning fork. 2. Automatic adjustment of the amplitude to the desired value. 3. Automatic adjustment of the phase difference between the amplitude and the applied force. 4. Visual indication of amplitude. 5. Visual indication of the phase difference. 6. Possibility of manual adjustment of the phase difference. These properties permit indirect measurement of the frequency at any moment making it possible to change tubes without making necessary a new calibration by comparison with a primary frequency standard.

INTRODUCTION

SINCE the dimension of frequency is the reciprocal of time, it is neither convenient nor necessary to adopt a primary frequency standard independent of time.

The universally adopted primary standard of time is the sidereal day; in this way, the primary standard of frequency will thus be determined by the frequency of the earth's rotation about its axis.

In reality, it has been experimentally demonstrated that the frequency of this movement is not constant in each instant; however, its mean value can be considered sufficiently constant. In speaking of radiofrequencies, the frequency of the primary standard (rotation of the earth) is very small with respect to these frequencies, so that it is necessary to employ an intermediate source of oscillations that will furnish a link between these two phenomena.

As a result, it becomes necessary to employ a secondary standard of frequency. Such a standard must have the following requisites:

1. In order to make comparison with a primary standard possible, the constancy of the frequency must be such that the number of oscillations produced in a time sufficiently large can be considered as n times larger than the number of oscillations produced in a time proportionately smaller.

2. It must be able to produce a frequency of the same order of magnitude as the frequency to be compared.

The ideal solution is a system consisting of an electro-mechanical oscillator whose frequency can be maintained constant for a long period of time and a frequency multiplier which can produce a band of discrete frequencies of the same order as those to be compared to it. Among the most-used mechanical oscillators, the following can be cited: magnetostriction oscillators; crystal-controlled oscillators; tuning-fork oscillators.

The magnetostrictive effect is normally little used for the control of generators because its use can be replaced by the two following methods with advantages. Of these two, both have advantages and disadvantages. In general, the crystal oscillator is used only in circuits for the production of frequencies greater than 50 kc.

In the case of a standard of frequency that must be compared with the frequency of rotation of the earth, this high fundamental frequency of oscillation is a disadvantage which makes it necessary to subdivide the frequency in order to arrive at a more workable value for comparison purposes.

On the other hand, if it is desired to obtain a discrete band of high frequencies of closely-

* Original text published in *Revista Electrontecnia* **27**, 383 (October, 1941).

spaced components it is necessary to multiply the frequency by the same order of magnitude as the subdivision formerly needed. Although, according to the work of Giebe and Scheibe,[20,36] Grützmacher,[55] Schuhmacher,[57] Wuckel,[66] and Beckmann,[67] much lower frequencies can be obtained with crystals oscillating transversally, in general it is not normally convenient to use them this way as the resulting temperature coefficient of frequency is very high (of order of 10^{-4} per °C). At the same time the influence of the constants of the circuit (plate potential, grid-cathode capacity, etc.) all add up to make it difficult to maintain the constancy of the frequency within the limits that can reasonably be expected of a standard generator.[29,30,33,41,49,50,62,68]

Moreover, as it is not possible to obtain by machining methods a frequency accuracy of better than 1 part in 10^5,[13,21,27,34,39] it is necessary to employ a mounting containing an adjustable air gap in order to obtain an exact frequency and thus avoid the inconvenience of fractions in the following multiplication.

Figure 1 (not reproduced herein), taken from the *Lehrbuch der Hochfrequenz Technik* by Vilbig, shows the variation of frequency with air gap. By examining this curve, it is seen that the control is very critical, thus requiring micrometer screw adjustments, metals of low temperature coefficients of expansion, all of which complicates the equipment to some extent.

Other authors adjust the frequency further, as permitted by the cut of the crystal, by making use of the effects of the circuit constants.

Figure 2 (not reproduced herein), taken from the work of Rohde and Leonhard[69] represents these effects.

In this case the control is obtained by the variation of C_g. This method also has its drawbacks as the frequency constancy depends upon the constancy of the capacity C_{g1}, needed to produce the desired frequency, and this capacity will be a function of temperature, pressure (change of the dielectric constant of the media in which C_g is immersed), etc. Because of these considerations the tuning fork was adopted for an electromechanical vibrator as it fulfills the following conditions:

1. One can obtain a frequency low enough to actuate a synchronous motor that is attached to a clock mechanism designed to indicate hours, minutes, and seconds so that comparison can be made with the primary frequency standard (frequency of rotation of the earth).

2. By using the proper material a very low temperature coefficient of frequency can be obtained.

3. It is not necessary to machine precisely in order to obtain a frequency represented by a whole number as this problem can be solved by adjusting the amplitude of oscillation. This does not mean that we must completely reject the possibility of using a quartz crystal as the control element in a frequency standard, depending on whether its employment facilitates or hinders, in any particular case, the solution of the above considered problems.

Suppose we have constructed an appropriate frequency standard, the next problem is to obtain an adequate number of spaced frequencies distributed over a sufficiently broad band. This can be obtained by using a system of frequency multiplication.[70–80] Among the various possible methods of frequency multiplication, the multiplier of Abraham and Block has been selected as it is considered to be the most stable and simple.

TUNING-FORK GENERATOR

To continue we shall study the features that must be incorporated in the tuning-fork oscillator in order to comply with the demands of frequency stability.

A tuning-fork generator can be represented diagrammatically as shown in Fig. 3.[83–89] The prong A, of the tuning fork forms part of the magnetic circuit of the coil B_1, whose core is a permanent magnet M_1. Upon vibrating, the tuning fork varies the air gap n, causing a corresponding change in the reluctance of the magnetic circuit of B_1.

This variation of the reluctance causes a corresponding variation in the flux traversing the core of the coil B_1, consequently generating a difference of potential e between its terminals. This potential is amplified by the amplifier V, which feeds the coil B_2, thus giving back to the oscillating system the energy it has lost, making it possible for the system to vibrate indefinitely.

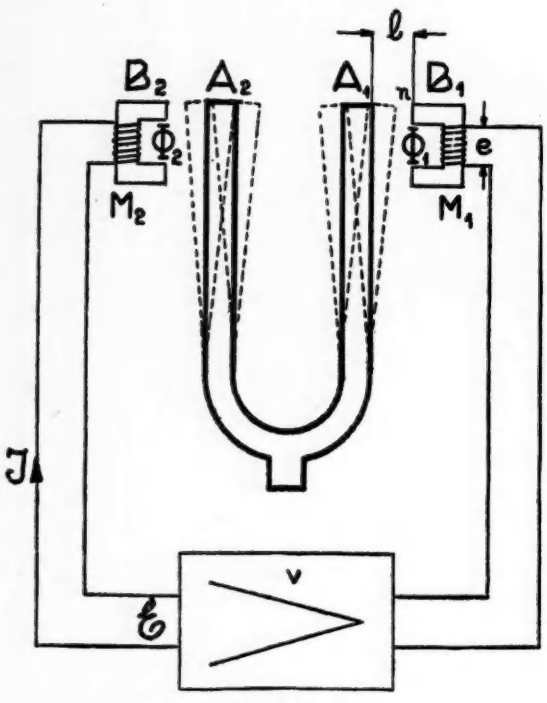


FIG. 3.

MATHEMATICAL STUDY

The mathematical study of this oscillation is directed toward becoming acquainted with the factors that influence the frequency.

Calling the maximum amplitude of oscillation X and considering that the amplitude is sufficiently small that the motion follows a sinusoidal law, the instantaneous amplitude will be

$$x = X \sin \omega t = \dot{X}. \tag{1}$$

The flux ϕ_1 passing through the coil will be, according to the law of Hopkinson

$$\phi_1 = \frac{k_0}{l-x} = \frac{k_1}{1-(x/l)\sin \omega t}. \tag{2}$$

Expanding into a series and limiting the expansion to the first two terms we will obtain

$$\phi_1 = k_1(1+(x/l)\sin \omega t \cdots), \tag{2'}$$

and the potential at the terminals of the coil will be

$$e = -k_1\omega(x/l)\cos \omega t = -k_2 X \cos \omega t = \dot{E}. \tag{3}$$

This potential is amplified, and the resulting amplified voltage $\dot{e}$ in general will not be in phase with $\dot{E}$. The potential $\dot{e}$ gives rise to a current j, which, in turn generates the flux ϕ_2, which will be proportional to the magnitude of $\dot{X}$, being a complex factor in order to account for the phase difference. That is to say

$$\phi_2 = \dot{X}(a_1 + jb_1). \tag{4}$$

a_1, like b_1, can be positive, negative, or zero as the case may be. This flux is superimposed upon the permanent flux ϕ_{02} of the magnet M_2, giving rise to a force

$$F = k_3[\varphi_{02} + \dot{X}(a_1 + jb_1)]^2 = k_3\varphi_{02}^2 + \dot{X}(a_2 + jb_2), \tag{5}$$

dropping terms of higher order.

Making

$$a_2/b_2 = \tan \phi, \tag{6}$$

the Eq. (5) can be written, considering only the oscillating term

$$\dot{F} = k_4 X \cos(\omega t + \varphi) = k_4 X[\cos \omega t \cos \varphi - \sin \omega t \sin \varphi] = k_4 \frac{dx}{dt} \cdot \frac{\cos \varphi}{\omega} - k_4 x \sin \varphi. \tag{7}$$

In the steady state, the motion will be given by the differential equation,

$$\alpha \frac{d^2x}{dt^2} + \beta \frac{dx}{dt} + \gamma x = k_4 \frac{dx}{dt} \frac{\cos \varphi}{\omega} - k_4 x \sin \varphi, \tag{8}$$

where α, β, and γ are the usual constants of the mechanical system. Equation (8) can be written

$$\alpha \frac{d^2x}{dt^2} + \left(\beta - k_4 \frac{\cos \varphi}{\omega}\right)\frac{dx}{dt} + (\gamma + k_4 \sin \varphi)x = 0. \tag{8'}$$

In order for this to be realized in the supposed steady state the following must hold.

$$\beta = k_4 \cos \varphi/\omega,$$

so that the system will oscillate with the angular velocity

$$\omega = \left(\frac{\gamma}{\alpha} + \frac{k_4}{\alpha}\sin \varphi\right)^{\frac{1}{2}} = \left(\omega_0^2 + \frac{k_4}{\alpha}\sin \varphi\right)^{\frac{1}{2}}, \tag{9}$$

in which it is observed that resulting frequency of the generator will depend on the natural frequency ω_0 of the tuning fork and the sine of

the phase angle between the amplitude of the fork and the force which is seen in Fig. 4, as a result of the vectorial interpretation of Eq. (8) for the case of negative φ (phase angle), the most frequent in practice. Considering that φ is the phase angle between the force and the velocity, in that which follows this angle will be considered and not the phase angle between the force and the amplitude. In the following discussion, however, we will consider the phase angle φ between the applied force and the velocity rather than the phase angle between the force and the amplitude.

FACTORS WHICH GIVE RISE TO FREQUENCY CHANGES

Previous work[89–101] has demonstrated that the following are the principal factors that affect the natural frequency of a tuning fork:

- Temperature
- Natural frequency of the resonance chamber
- Damping
  - External magnetic fields
  - Density of the medium
  - Method of coupling energies to external circuit
- Amplitude

If the tuning fork is electromagnetically excited it will also be necessary to consider the influence of φ (Eq. (9)).

Continuing, we shall analyze each of these factors.

TEMPERATURE

Past studies[89, 90, 99] have shown that variations of temperature are the largest factor affecting the frequency of the tuning fork.

According to the work of Konig,[93] Robin,[100] and other investigators, the frequency of a tuning fork is given by the empirical formula

$$f=K(E/D)^{\frac{1}{2}}(e/l^2), \qquad (10)$$

where E = the modulus of elasticity, D = the density of the material, e = thickness, and l = the length of the tines (prongs).

From the study of this formula one can deduce that temperature affects frequency by acting upon the modulus of elasticity and the geometrical dimensions of the tuning fork.

Calling α the linear coefficient of expansion, β the temperature coefficient of frequency, and γ the temperature coefficient of the modulus of elasticity, one can immediately derive the following relation between these coefficients from expression (10)

$$\beta=\tfrac{1}{2}(\alpha+\gamma). \qquad (11)$$

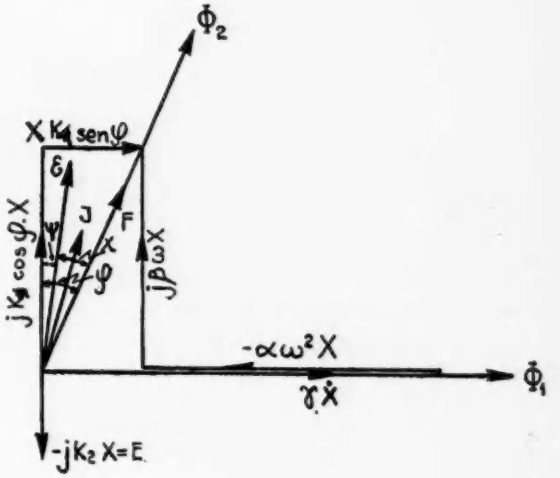


FIG. 4.

By a suitable selection of material, the signs of α and γ can be made such that they cancel, making $\beta=0$.

The works of Robin,[100] Chevenard,[102] and of Guillaume[103] serve to show that a mixture of varying amounts of Ni and Cr in steel varies the temperature coefficient of the modulus of elasticity, and thus they arrive at an alloy they call "Elinvar," whose principal characteristic is a low temperature coefficient of the modulus of elasticity; most of the precision tuning forks made by the firm Sullivan, of London, have been made of such material. From the careful survey by Namba,[83] the persisting temperature coefficient of frequency drops to about the order of 1.8×10^{-5} which is surely a great advance over ordinary tuning-fork steel whose value varies about the value 3×10^{-4}.[90, 100]

Figure 5 taken from the work of Chevenard[102] shows how the temperature coefficient of the modulus of elasticity varies with the variation of the composition of the alloy.

Based on this data, Fahlenbrach and Meyer, of the Krupp firm of Essen, have investigated a special tuning-fork steel, arriving at a type which they designate WT-11[104] whose relative variation of frequency with temperature for different heat treatments is shown in Fig. 6.

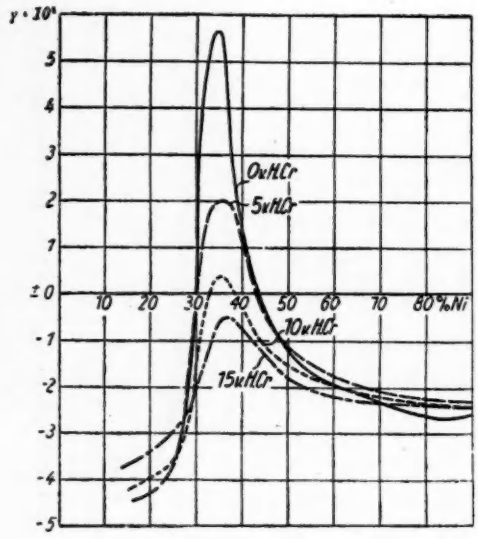


FIG. 5.

This treatment consists first of a tempering at 1000°C with quenching in water followed by annealing with quenching in air.

Among the treatments shown, the third is the best, yielding a material whose temperature coefficient is 1×10^{-6}.

For interpolation (see Fig. 7, not reproduced herein) it is seen that the optimum treatment will be obtained with an annealing at 685°C, with which a zero temperature coefficient should be obtained. In practice one generally obtains a temperature coefficient of about 2×10^{-7}, which can be considered linear between 0° and 70°C. Thus it has arbitrarily been decided to work at 35°C.

CONSTANCY OF TEMPERATURE

The value of the temperature coefficient makes it necessary to place the tuning fork in a carefully designed constant temperature chamber in which the ambient temperature changes are less than .01°C.

In general, a constant temperature box consists of a chamber, thermally insulated from the surroundings, inside of which a thermostat element is placed to control automatically the supply of heat thus smoothing out the small variations about a given temperature.

In the present case a mercury thermostat has been used, with the contacts adjusted for 35°C in accordance with the preceding considerations. The difference of temperature necessary to operate this thermostat is 0.1°C, thus the temperature of the bulb oscillates between 35.1°C and 34.9°C during operation, thus we evidently cannot place the tuning fork in its vicinity if we wish to maintain the temperature variations within the limits formerly arrived at (0.01°C).

Consequently, it is necessary to place an adequate thermal system between the constant temperature chamber and the control element so that the temperature variations reaching the chamber are small compared to those existing about the control element.

This is accomplished by using an attenuating layer of suitable material.

The variation of temperature ΔT of a point near a wall of infinite surface situated at a distance l from a temperature variation ΔT_0 is given, according to Marrison[105] by the expression $\Delta T=\Delta T_0 e-(DH\omega l/2C)^{\frac{1}{2}}$, where D=density of the material of the wall, H=its specific heat, C=thermal conductivity, and $\omega=2\pi F$, where F=frequency of the temperature variation.

Evidently, from this formula, the efficiency of an attenuating layer will be greatest when the expression $(DH/2C)^{\frac{1}{2}}$ is at a maximum. For the most used materials, the values of the density, specific gravity, thermal conductivity, and the values of the latter quantity can be found tabulated in certain works.[105, 108–110]

Taking into account the ease with which it is obtained (in Germany) Kieselgur has been adopted with a layer of 2 cm thickness, and 1/60 as the frequency of operation. The resulting

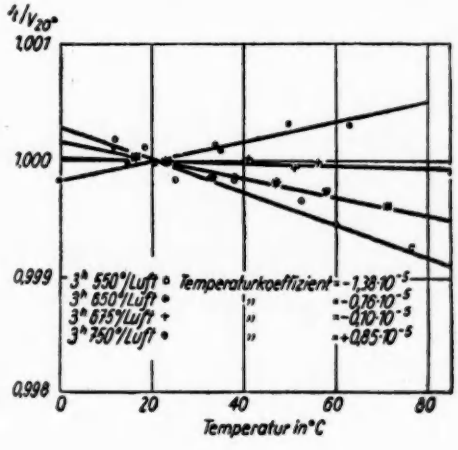


FIG. 6.

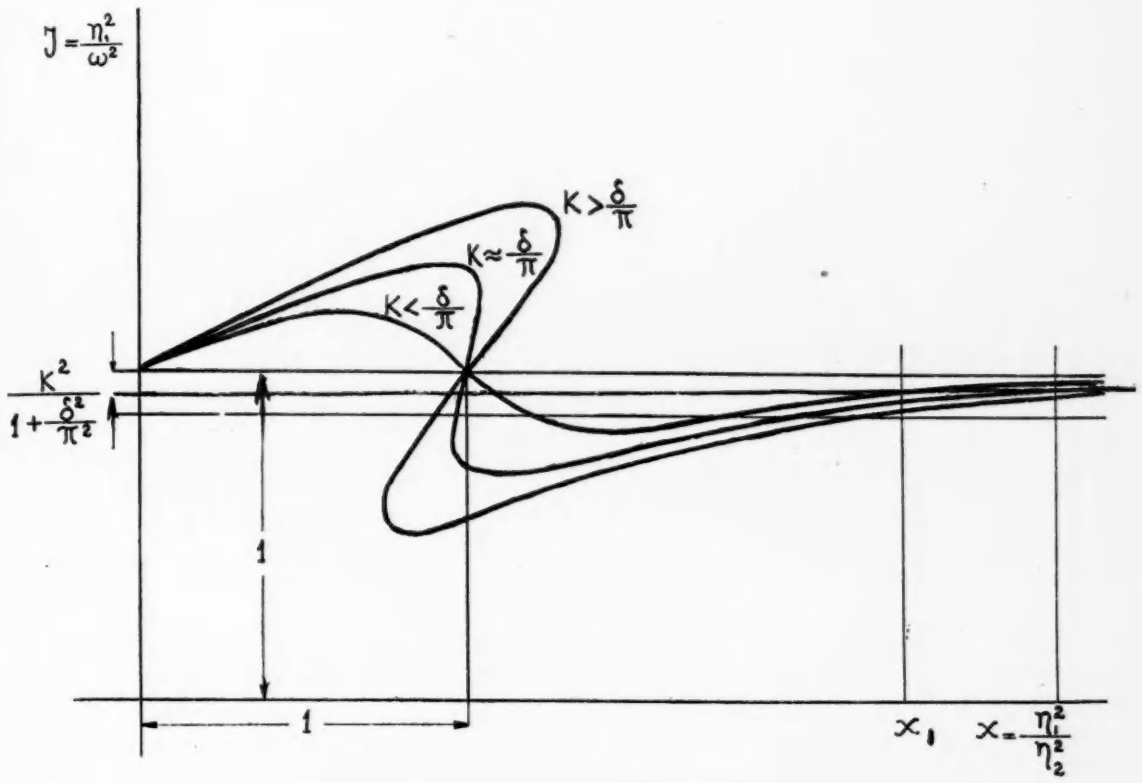


FIG. 9.

temperature variation that penetrates to the interior is

$$\Delta T = 0.1e^{-7} \approx 0.0001°\text{C}.$$

With the purpose of securing as uniform heat distribution as possible, the constant temperature chamber has been made cylindrical in shape. The heating element was made of Nichrome wire of appropriate diameter, wound on the cylindrical surface. The spacing of the winding is not constant along the whole length of the support, since in the ends the number of turns per unit length is increased in order to prevent radiation cooling of the ends, which are not heated.

Between the heating element and the attenuating layer a distributing layer has been placed in order to iron out the small differences in temperature that can be caused by unequal heating of neighboring points on the surface of the cylinder. Consequently, this layer must have a higher thermal conductivity than the wall and a lower specific heat. These qualities are sufficiently approximated in aluminum. The thermostat has been placed over the distributing layer and in intimate contact with it.

A cylinder of aluminum has been placed inside to help equalize the small differences of temperature caused by uneven conduction of the attenuating layer which is occasioned by inhomogeneity (from the thermal point of view) in the Kieselgur (greater pressure on the lower part, more moisture in the top part, etc.).

This cylinder also serves to support the tuning fork and the excitation and grid coils.

The dimensions of the resonance chamber have been chosen so that it forms a damped oscillator system coupled to the tuning fork.

A study of this system with regard to the effect of the resonance chamber upon the frequency of the tuning fork gives an equivalent electrical circuit composed of a generator coupled to a loading circuit.

The influence of the natural resonance frequency of the load circuit upon the resulting frequency of the combination is given by the

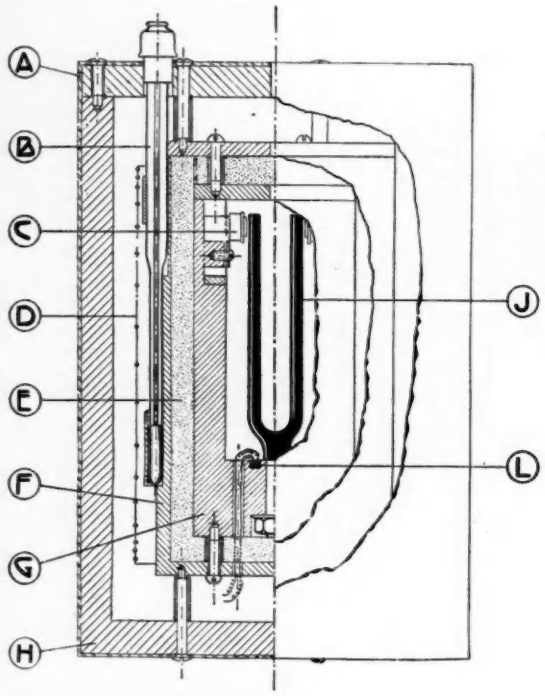


FIG. 10.

A. Magnetic shield
B. Thermostat
C. Exciter coil
D. Heating element
E. Kieselgur
F. Outer distributing cylinder
G. Inner distributing cylinder
H. Thermal insulator
I. Tuning fork
J. Electric thermometer resistance

expression (111)

$$(1-y)/(1-y/x)=k^2/[d^2y+(1-y/x)^2],$$

where

$$x=\eta_1^2/\eta_2^2, \quad y=\eta_1^2/\omega^2, \quad d=\delta/\pi,$$

η_1 = natural frequency of the generator, η_2 = natural frequency of the load circuit, ω = the resulting frequency (of combination), and δ = logarithmic decrement of η_1 in the 2nd circuit.

Figure 9 graphically represents the function $y=f(x)$ taking the coefficient of coupling k as the parameter.

These results have been verified experimentally for the case of tuning forks coupled to resonant chambers by König.[93]

Figure 9 reveals that whatever the coefficient of coupling may be, whenever the natural frequency of the box approaches closely the frequency of the tuning fork $\eta_1 \approx \eta_2$, a small variation of η_2 brings about a large variation of ω.

Anticipating some possible variation of the natural frequency of the resonance chamber as well as of the coefficient of coupling k (variations in the tuning fork support) with the resulting variations in ω, the resonance chamber has been designed of such dimensions that the relation $x_1=\eta_1^2/\eta_2^2$ is very large as indicated in the above figure. The presence of the Kieselgur included between the two aluminum cylinders indirectly contributes in a large way to this condition as it greatly augments the logarithmic decrement δ of the resonance chamber, giving with the chosen working conditions,

$$\omega=\eta_1, \quad (\eta_1^2/\omega^2=1),$$

since

$$k^2/(1+\delta^2/\pi^2)\approx 0 \qquad \text{(see Fig. 9)}.$$

The heat chamber is completed by an insulating layer of balsa wood, encompassed by a cylinder of iron of high magnetic permeability in order to shield the tuning fork from external magnetic fields. This cylinder is carefully polished on the surface to prevent the radiation of heat.

Figure 10 represents the construction of the constant temperature chamber. The temperature distribution is shown in Fig. 11.

All possible has been done to accomplish the condition of temperature symmetry, even in the ends, by making the top equal to the bottom in reference to the spacing of the insulating material, distributor, attenuators, and air.

Finally, in order to protect the mercury-platinum contact of the thermostat, a relay was placed in the plate circuit of a triode, whose grid potential is controlled by the thermostat. Thus the heating current flows only through the contacts of the relay as shown in Fig. 12.

With the object of minimizing the effect of the small magnetic field set up by the heating ele-

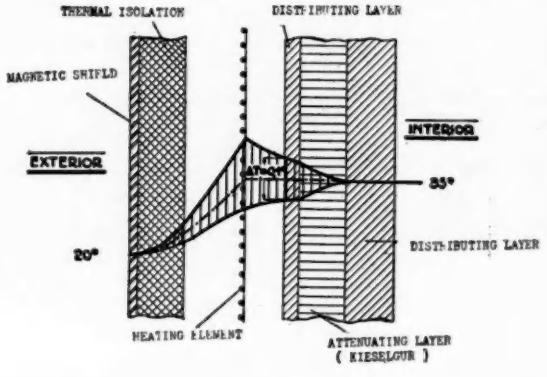


FIG. 11.

ment, alternating current was used, so that the distributing cylinders furnish an efficient shield for the varying magnetic fields.

The current in the heating element has been chosen in such a way that for an external temperature of 20°C the relay contacts are closed half of the time, as it has been shown experimentally that this is the best operating condition for the thermostat.

Under these conditions the apparatus consumes 60 watts of power for an external temperature of 20°C.

Figure 13 (not reproduced herein) shows a detailed view of the important parts of the constant temperature chamber before assembly.

Figure 14 (not reproduced herein) shows a view of the assembled chamber, with the Kieselgur already assembled.

In order to detect any faults in the thermostat that would interfere with the constancy of the temperature, an electric thermometer was placed in the constant temperature chamber in the form of a Wheatstone bridge composed of three nickel and one iron resistance. The value of the resistances were chosen in such a manner that balance was obtained at exactly 35°C. In this manner the indication of the instrument is independent of the potential fed to the bridge (bridge in equilibrium). This necessitates the use of a galvanometer as the indicating instrument with the zero in the center of the scale.

By choosing the values of the resistances carefully, this system is very sensitive, allowing observations of temperature differences as low as 10^{-3}°C.

INFLUENCE OF AMPLITUDE AND PHASE DIFFERENCE ON FREQUENCY

Formula (9) shows the effect of frequency upon the phase difference between the velocity and the driving force acting on a mechanical oscillator.

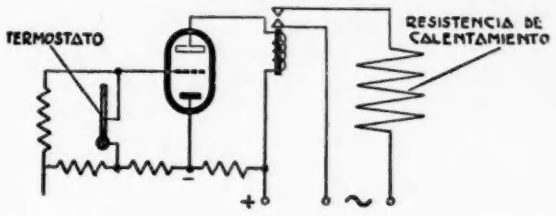


FIG. 12. Thermostat (left side of figure). Heating element (right side of figure).

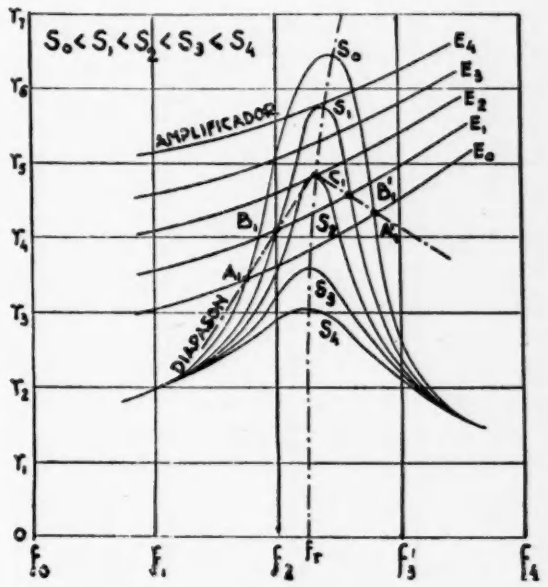


FIG. 16. *Ordinate*—
ratio of output/input voltage of tuning fork.
ratio of input/output voltage of amplifier.

Note: Diapason—tuning fork.
Amplificador—amplifier.
Entrada—input.
Salida—output.

Following the experiences of König,[93] Miller,[113] and Dye[89] who have shown that the frequency also depends upon the amplitude of oscillation, a variation of 10 percent in amplitude of the adopted tuning fork would cause a corresponding frequency change of the order of 10^{-8}.

When the oscillations of the tuning fork are excited by a valve amplifier one can study the influence of these factors on the frequency by applying the method which has been revealed by Horton[114] in the case of valve oscillators.[115]

The set-up of amplifier-tuning fork can be represented in a simplified form as shown in Fig. 15 (not reproduced herein). Considering only the tuning fork and the excitation and generator coils, it is evident that the system will act like an attenuator (either positive or negative) according to the constants of the system (number of turns on the coils, characteristics of the magnetic circuit, etc.), thus to each potential applied to the excitation coil there will be a corresponding potential in the generator coil. Evidently if we want to obtain a constant generator-coil potential, the excitation coil potential will be a function of frequency as shown in Fig. 16.

R, the ratio of the potentials in the generator and excitation coils, has been taken as the ordinate, the frequency as abscissae, and the generator potential S as parameter. It is to be noted that for a given frequency the ratio R decreases with an increase in the generated potential; as the increase in this potential signifies an increase in the amplitude which appears as a larger damping coefficient (greater mass of air in movement, etc.).

Analogously, applying a given potential to the input of the amplifier, the output potential, in the most general case will be a function of the frequency and amplitude of the input voltage (non-linear characteristics). These curves have been shown in Fig. 16, taking as ordinates the ratio of the input to output potential, the frequency as abscissae and the input potentials as parameter. E_0, E_1, E_2, E_3, and E_4 equal the outputs of the tuning fork S_0, S_1, S_2, S_3, and S_4, respectively.

It is evident that once the steady state has been reached, the points of operation must be those such as A_1, B_1, C_1, found on the curve, A_1, B_1, C_1, B_1', A_1', so that the output of the amplifier is applied to the input of the tuning fork and the output of the tuning fork is applied to the input of the amplifier.

On the other hand the variation of the phase difference Ψ between the input and output voltages to the tuning fork have been represented in Fig. 17 as a function of frequency for the same output potentials S, considered in the last figure. Again in this case it has been shown that the

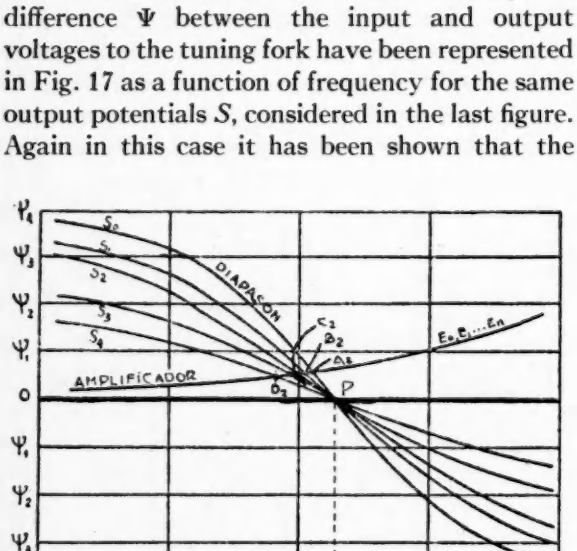


FIG. 17.

damping (resistance component of the mechanical impedance) increases with amplitude.

On the same scale, the difference of phase between the input and output potentials of the amplifier have been represented, which in accord with theoretical prediction and experimental results can be considered independent of the input potential E.

In the steady state, the potential between the points a, b (Fig. 15) must have the same phase with respect to the points C, D whether we are considering them with respect to the amplifier or the tuning fork.

Thus the possible points of operation in this regard will be those such as A_2, B_2, C_2, D_2.

In order to find the one point of operation that satisfies both the condition of amplitude and of phase simultaneously, we must find the curve linking the output potential of the oscillator with the frequency for the possible modes of vibration (Points A_1, B_1, C_1, A_1', B_1', etc., Fig. 16), so that each point of this curve satisfies the condition of amplitude (input voltage of amp. = output voltage of tuning fork, for voltage input to tuning fork = output voltage of amplifier).

Suppose now that the difference of phase between the input and output potential of the amplifier is varied, keeping the amplitude relation constant (curve A_1, B_1, C_1, B_1', A_1'). This alters the curve A_1, B_1, C_1, D_1, so that it will cut $A_1, B_1, C_1, B_1', A_1'$ at a point of different abscissae f_f, causing a corresponding change in frequency.

The only way of avoiding this effect is to arrange things in such a way that the curve A_2, B_2, C_2, D_2, will be a straight line parallel to the ordinate axis (f_f = constant) for any value of the other parameters.

This condition can be fulfilled only if the straight line representative of the variation of the difference of phase of the amplifier cuts the one for the tuning fork at the point P which, as we see by observation of Fig. 17, is that one for which the only possible frequency of oscillation is the resonant frequency f_r.

It must be realized that the angle Ψ, to which Fig. 17 refers, involves the difference of phase ϕ between the driving force and the velocity, and the phase difference x between the voltage applied to the excitation coil and the flux generated in its magnetic circuit for which the

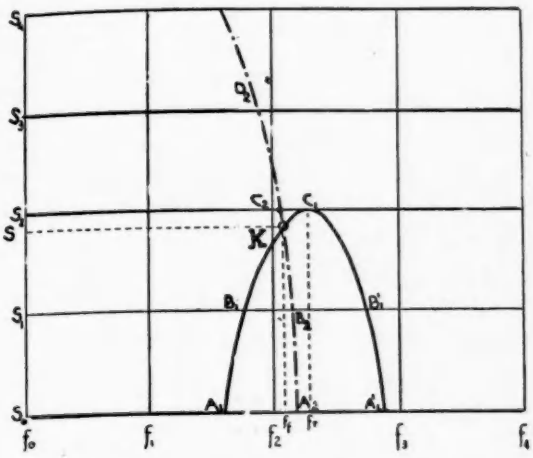


FIG. 18. *Ordinate*—output voltage of tuning fork.

expression is

$$\Psi = \varphi + x \qquad \text{(see Fig. 4)}.$$

This fact does not in general affect the problem if the natural frequency of the excitation coil is very different than that of the tuning fork. This can be accomplished in practice by using a small number of turns and by having small distributed capacity. In this case the curves representative of the phase difference φ between the force and the velocity as a function of frequency can be represented by the system of coordinates of Fig. 17 only by translation parallel to the axis of the abscissae of $-x$. In the case where the losses of the excitation coil depend on the input potential, the ordinates corresponding to particular frequencies will also be changed in value, but in whatever manner the point p, intersection of particular curves, always maintains the abscissa f_r. This is equivalent to saying that the tuning fork will oscillate on its natural frequency only if $\varphi = 0$.

This result agrees perfectly with that obtained by purely analytical methods (see Eq. (9)).

If curves A_2, B_2, C_2, D_2, of Fig. 18 are considered constant, a change of the amplification factor of the amplifier (change of plate potential, filament current, etc.) or the damping coefficient of the mechanical oscillating system (variation of the density of the medium in which the vibrator is submerged, influence of external magnetic fields, etc.) will produce a corresponding change in the curves A_1, B_1, C_1, B_1', A_1' (see Fig. 16) giving as a result a point of intersection K' with A_2, B_2, C_2, D_2 at a different value of abscissae f_f than the previous one. But such a thing does not occur if the curve A_2, B_2, C_2, D_2, is a straight line parallel to the ordinate axis ($\varphi = 0$) as indicated in Fig. 19.

From the preceding consideration the following conclusion results:

If the phase difference between the driving force and the velocity of the moving system is zero, changes in the amplitude of oscillation will not influence the frequency. This conclusion was also implicitly contained in Eq. (9).

In practice this result is not observed because the force necessary to deflect the oscillating mass from its equilibrium position is not a linear function of X (deflection) as we have assumed, but a polynomial of degree n.

That is to say,

$$Fe = \gamma X = aX + bX^2 + cX^3 + dX^4 + \cdots, \quad (12)$$

where a, b, c, d, are constants and the other symbols have the same significance as in Eq. (9). Taking out the common factor X we obtain

$$\gamma X = [a + bX + cX^2 + dX^3 + \cdots]X, \quad (13)$$

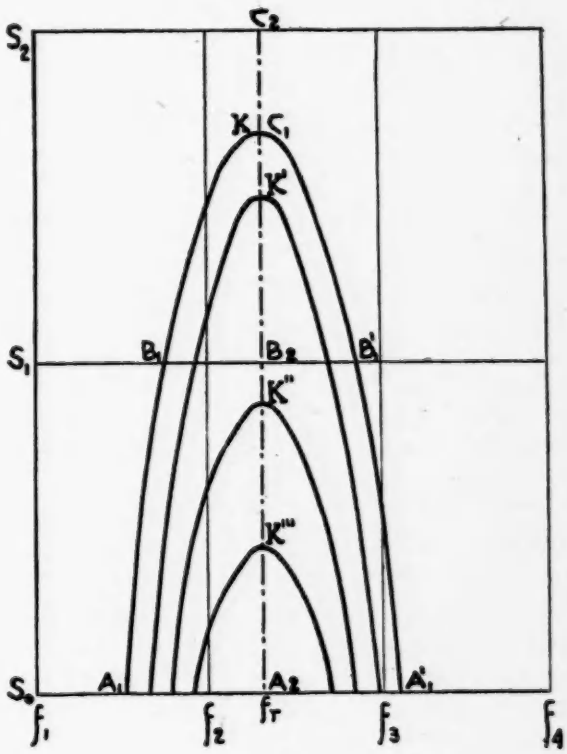


FIG. 19.

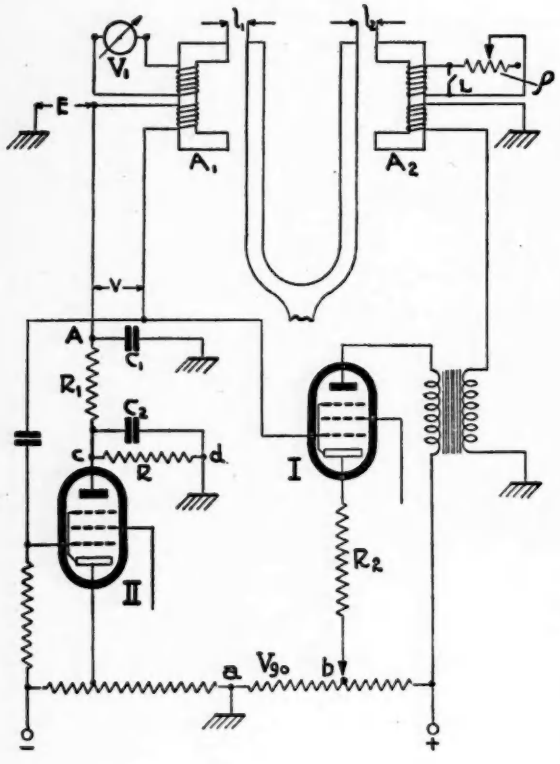


FIG. 20.

and since $\omega_0^2=\gamma/\alpha$ there will finally result

$$\omega_0^2=\frac{a+bX+cX^2+dX^3+\cdots}{\alpha}. \qquad (14)$$

That is to say that the resonant frequency depends on the amplitude of oscillation and this dependency will become smaller as the oscillating mass α becomes larger. As a consequence it is best to make the tuning fork with the largest possible dimensions, being careful to retain the necessary relation between the length and spacing of the prongs, in order to obtain the desired frequency.

From the preceding considerations comes the necessity of constructing an amplifier in such a way that it will furnish just sufficient energy to maintain a constant amplitude of oscillation under all possible working conditions (varying supply voltages, varying damping factors in the oscillator, etc.) and moreover make the phase difference between the force and the velocity zero.

AUTOMATIC REGULATION OF AMPLITUDE

The first condition can be obtained with the amplifier shown in Fig. 20, which consists of a variable μ-amplifier tube I and an auxiliary tube II.

Considering only tube I, for a constant input potential the output potential will depend upon the operating bias V_{g0} (variable μ tube) if the rest of the circuit constants remain fixed.

Upon introducing tube II, whose operating grid bias has been chosen near the cut-off point, it will generate pulsating direct current across the terminals of the resistance R, which depends on the potential applied to the grid by the given working conditions as seen in Fig. 21 with reference to the curves $Ia-Va$.

Filtering this voltage with a filter consisting of condensers C_1 and C_2 and the resistance R_1, a d.c. potential E will be obtained at the point A which can be considered, in the first approximation, proportional to the maximum value of the a.c. potential applied to the grid of tube II, which in turn is proportional to the amplitude of oscillation of the tuning fork. This potential provides the operating bias for tube I, controlling the amplification factor under these conditions, if the amplitude tends to diminish because of a change in the electromechanical constants of the system, the operating grid bias will be lowered increasing the amplification factor and thus the amplitude will tend to increase until it reaches a value close to the original. The reverse occurs if the amplitude tends to increase.

A Telefunken EFM11 was used as an amplifier

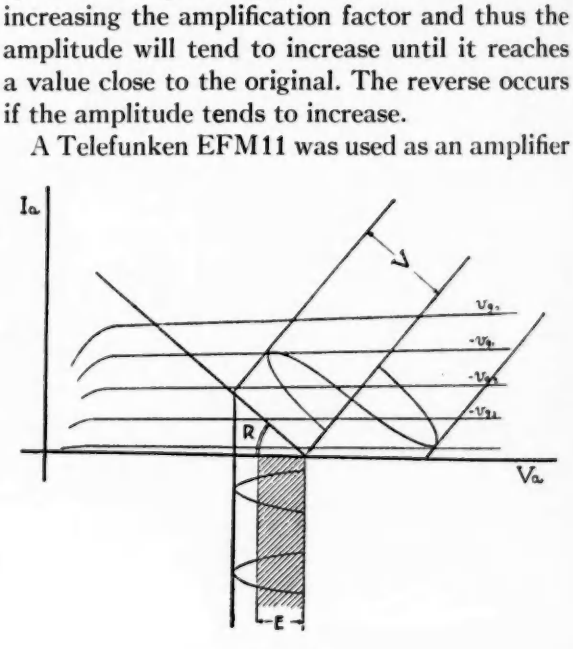


FIG. 21.

tube. It is a low frequency variable μ pentode provided with a visual tuning indicator. This tube was chosen because of its characteristics.

A special consideration was the constancy of the plate resistance for different values of operating grid bias. This is a very valuable condition, as we shall see in considering the effect of the change of phase difference φ between the velocity and the force. Figure 22, which we take from Scheel,[116] represents the change in the amplification factor as a function of the operating grid potential.

A variation of the amplification factor of 5/1 can be obtained for a variation of the grid potential between -1 and -20 volts.

In order to obtain greater sensitivity of regulation, the cathode of tube I has been connected to a point of negative potential with respect to ground so that the grid potential is the result of the difference between the potential is the result of the difference between the potentials *ab* and *cd* (Fig. 20).

In this manner the amplification factor is changed immediately whenever the amplitude varies even a slight amount from its normal value.

This circuit makes it impossible to start up without previously exciting the tuning fork by an auxiliary method, because otherwise, as no potential is generated at the terminals of the coil A, no current flows in the resistance R_1 and consequently the grid of tube I is placed at a high positive potential *ab* with a corresponding high plate current, which can destroy the tube. In reality this effect is almost completely annulled by the resistances R and R_1 which give the grid a negative bias as soon as current begins to flow through them. Without doubt, the effect of these resistances is not enough to prevent the grid potential from rising to a value of approximately 0, but what is more important, the effect can be completely avoided by placing a resistance R_2 in series with the cathode, which due to the plate current flowing, will bias the grid negative. Thus the oscillations can be automatically initiated without the necessity of exciting the tuning fork artificially. This is a great advantage if it is recalled that the tuning fork is placed in the constant temperature chamber which makes access from the outside difficult.

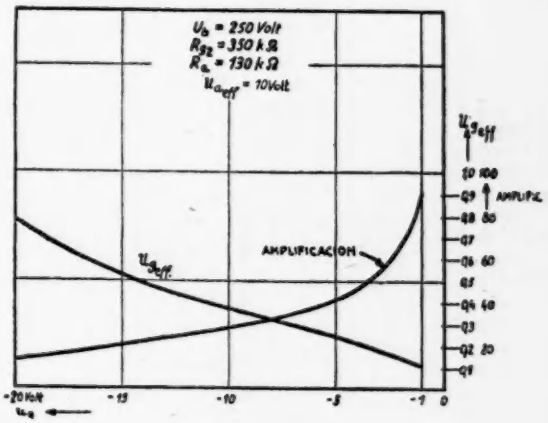


FIG. 22.

The resistance R_2 fulfills another important function. Assuming that the amplification factor is proportional to the amplitude of oscillation (effect of the automatic regulation) and taking into account the negative drop across R_2, the differential equation of the instantaneous amplitude, if the phase difference between the velocity and the force is zero, will be

$$\alpha\frac{d^2x}{dt^2}+\beta\frac{dx}{dt}+\gamma x=k_0(X-k_1R_2)\frac{dx}{dt}, \qquad (15)$$

where the symbols have the same significance as before.

This expression readily transforms into an expression of the form

$$x''+\epsilon\varphi(x)x'+\omega_0^2x=0, \qquad (16)$$

in which x' and x'' represent respectively the first and second derivatives of x with respect to t; $\omega_0^2=\gamma/x$; ϵ a constant and $\varphi(x)$ a function of the amplitude.

In this case

$$\epsilon=\frac{\beta+k_1k_0R_2}{\alpha} \quad \text{and} \quad \varphi(x)=1-\frac{k_0}{\beta+k_1k_0R_2}.$$

The solution of Eq. (16) from an analytical standpoint is rather difficult, but a graphical method can be employed as worked out by Van der Pol,[117] Lienard,[118] and Usui.[119,120] Figures 23–25 represent the results of this solution.

They refer to the variation of amplitude in the transient state just after having produced a sharp change in the loss factor β. Before the

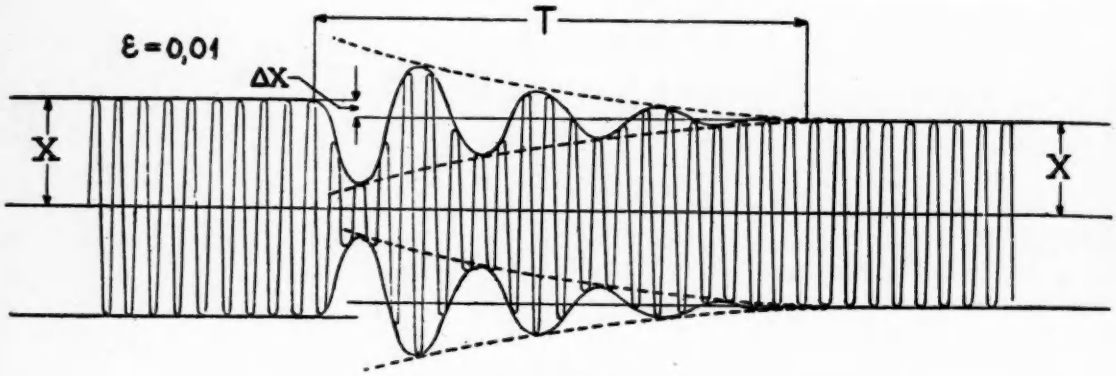


FIG. 23.

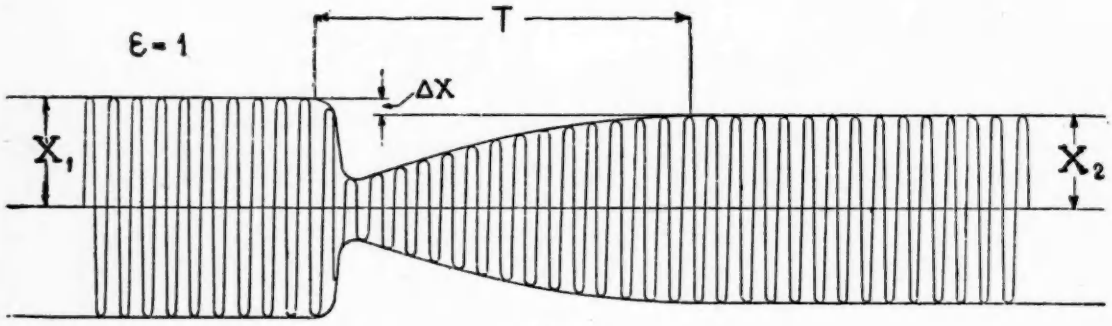


FIG. 24.

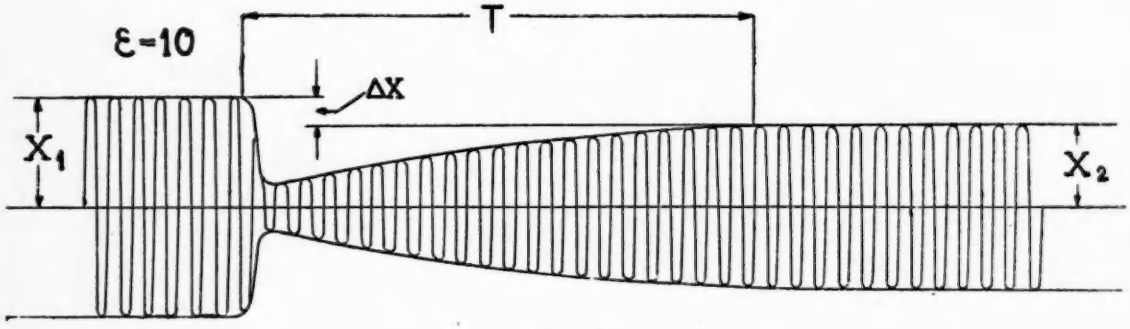


FIG. 25.

perturbation the maximum amplitude is X_1; the perturbation is produced with the resultant variations as shown during the time T_1, finally reaching a steady state X_2 which because of the basic principle of the amplitude control must be somewhat smaller than X_1.

It is evidently an advantage to reach the steady state X_2 in such a way as to avoid long periods of changing frequency. In conformity with this, it will be necessary to choose the value of ϵ in such a manner that the law that determines the variation of maximum amplitude during the transient period is aperiodic, which is found very close to the limit of the oscillating law (condition of T minimum). ϵ can vary between extensive limits set by the resistance R_2. These theoretical results have been verified experimentally, measuring the amplitude by means of the instrument and producing a sharp increase in the loss factor.

For this purpose a short circuit is made by the use of a key on an auxiliary winding on the same core as A_2. (See Fig. 20.)

The experimental results are in perfect agreement with the theoretical predictions.

Figure 26 represents graphically the time T

necessary to reach the steady state as a function of R_2.

The minimum time T corresponds to $R_2=2000$ ohms in this case.

In practice, it is hardly probable that such a sharp change of forms as indicated will occur, so that the automatic control of amplitude works under much more favorable conditions (slow variations of the electromechanical constants).

The effectiveness of the automatic control of amplitude has been verified by measuring the potential generated in the auxiliary coil wound over A, with an instrument of such sensitivity that full scale deflection was obtained (in this case 100 div.) when the tuning fork vibrated with normal amplitude.

As we have seen before

$$E_{\text{eff}}=kX,$$

taking logarithmic derivatives there results

$$\Delta E_{\text{eff}}/E_{\text{eff}}=\Delta X/X=\Delta\alpha/\alpha.$$

In this case the smallest value that could be observed on the instrument scale is $\Delta\alpha=0.2$ div. and since $\alpha=100$ div., $\Delta X/X=0.02/100=2\times10^{-3}$. That is to say, it would be possible to observe a change in amplitude whenever it is larger than $2\times10^{-3}X$.

This variation in amplitude corresponds to a change in frequency of $\Delta F=2\times10^{-10}$.

The verification of the regulation can be effected by producing a variable electromagnetic effect by producing a variable electromagnetic loss. To do this the circuit of the auxiliary coil A_2' is completed by the use of a variable resistance ρ.

Varying this resistance from 0 to ∞ causes a variation in amplitude measured as $\Delta\alpha=0.6$ div. which must correspond to

$$\Delta F=6\times10^{-10}.$$

The same experiment using an amplifier without automatic control gave for $R=50{,}000$ ohms

$$\alpha=\alpha_{\max}/2.$$

The system ceased to function at $R=20{,}000$ ohms.

Other experiments were tried such as the introduction of a prism of spongy gum between the prongs of the tuning fork, variation of the supply voltages, introduction of a magnetic shunt on the core of A_2, etc. All of which revealed perfect action of the automatic amplitude control.

FINE CONTROL OF FREQUENCY

Recalling the law that relates amplitude to frequency (Eq. (14)) one can obtain an additional advantage from this system. This is the possibility of obtaining a fine adjustment of frequency by regulation of the amplitude. This result can be obtained by using a variable potentiometer b (Fig. 20) which controls the operating potential (bias) of the grid. Each value of this potential determines a point of equilibrium for the system and a corresponding amplitude of oscillation results. Once adjusted this amplitude remains constant, to a good approximation, due to the action of the automatic control. This property is made use of to obtain the last degree of control of the natural frequency of the tuning fork which could not be obtained in the construction of the tuning fork by varying the mechanical dimensions. In the given case it was possible to make a total variation of the amplitude measured as $\Delta\alpha=25$ div. which corresponds to $\Delta F=10^{-7}$[$\Delta F=\varphi(\Delta\alpha)$ not linear according to Eq. (14)]. This range of variation can be sufficient in the case of a very carefully constructed tuning fork.

With great care, the geometrical dimensions of the tuning fork can be made so that the natural frequency approaches a whole number to

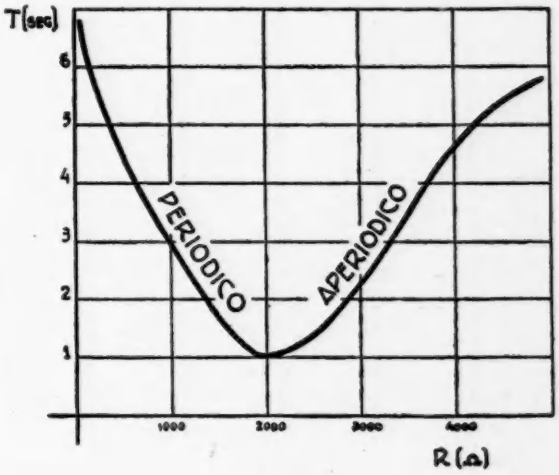


FIG. 26.

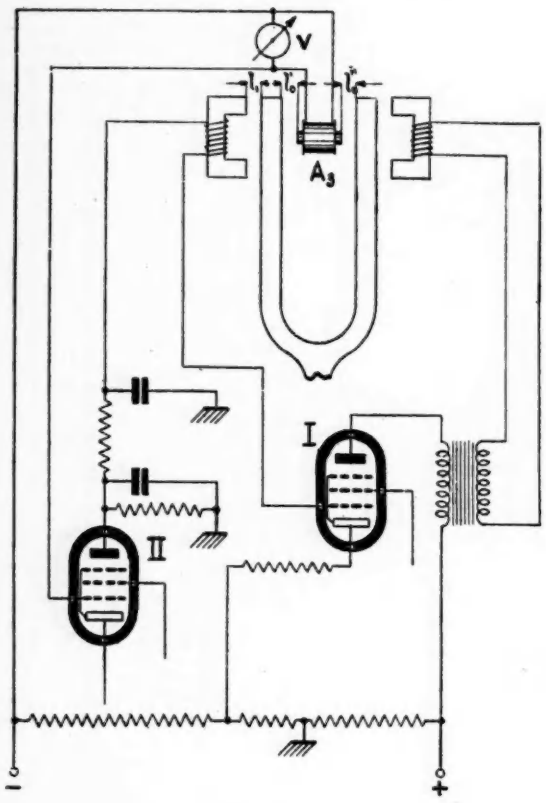


FIG. 27.

a good approximation; in this case 1000 c.p.s., and thereafter for effective measurements the final adjustment can be made by varying the amplitude. The necessary adjustments can be readily made during the course of the construction, assuring the preparation of a generator of exactly 1000 c.p.s. normal frequency by a procedure to be described.

Two oscillators are to be connected in such a way that the beat between them is made perceptible and the frequency of this beat will be made zero by adjusting the amplitude of oscillation. When this is attained, it will be necessary to note the position of the needle of the instrument that measures the amplitude on the α scale and take the precaution of always returning to this position after changing tubes, etc., with which it is assured that the amplitude remains constant within 2×10^{-3}.

The proposed system of automatic regulation also compensates a corresponding variation of amplification, the variations of the air gap l_2 or of the magnetic flux of A_2. Without doubt, if the variation of amplitude arises from a variation of l_1 the effect of the automatic regulation may not only be inefficacious, but may reach such a state as to be damaging. In effect, if we suppose that l_1 increases and that the amplitude of oscillation remains normal, the potential V generated in A_1 diminishes, which appears as an increase in the amplification, with an increase in the amplitude until the generated potential V' will be very close to V. The increase in amplitude causes a corresponding change in frequency which is not evident because the measured potential V_1 indicates the normal amplitude. In order to avoid this effect a modification can be made as shown in Fig. 27. The modification consists of a coil A_3 between the tines of the tuning fork. The amplitude indicating voltmeter is connected across this coil and also the grid of the auxiliary tube II.

Under these conditions an increase of l_0' is accompanied by a decrease in l_0'' and *vice versa* so that the generated potential is not changed, in the mean time the amplitude remains constant. The automatic control thus also permits compensation for variations of l_1. Without doubt, with the given system of mounting the coils that was adopted (see Figs. 10 and 13) a variation of l_1 is not probable, for it is possible to obtain very good results with even the disposition shown in Fig. 20, which has the advantage of being more easily attained.

AUTOMATIC CONTROL OF THE PHASE DIFFERENCE

The advantages to be derived from maintaining zero phase φ between the driving force and the velocity of the tuning fork has already been demonstrated. This phase difference depends upon the electrical and mechanical constants of the system which in turn are functions of many factors, which, in general, vary with the time (supply voltage of the amplifier, damping force of the oscillating system, etc.) so that it is evidently necessary to design an automatic control whose action will compensate for the possible variations of φ during operation.

In order to effect this control, the plate circuit of tube I (of Fig. 20) has been changed as shown in Fig. 28 by introducing a new auxiliary tube III.

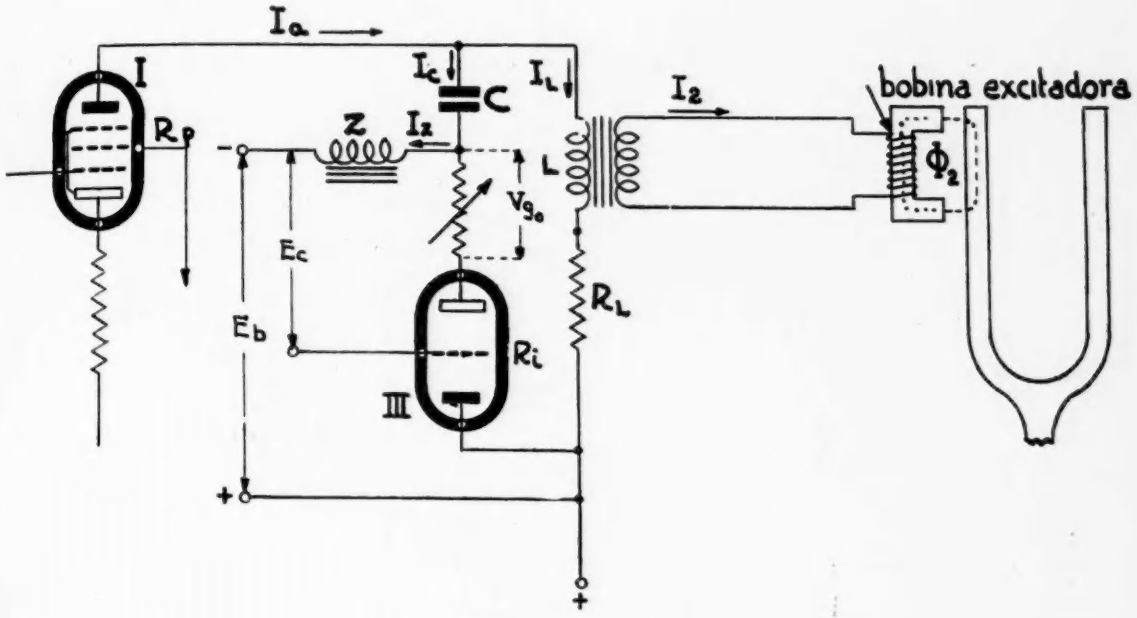


FIG. 28. Exciter coil.

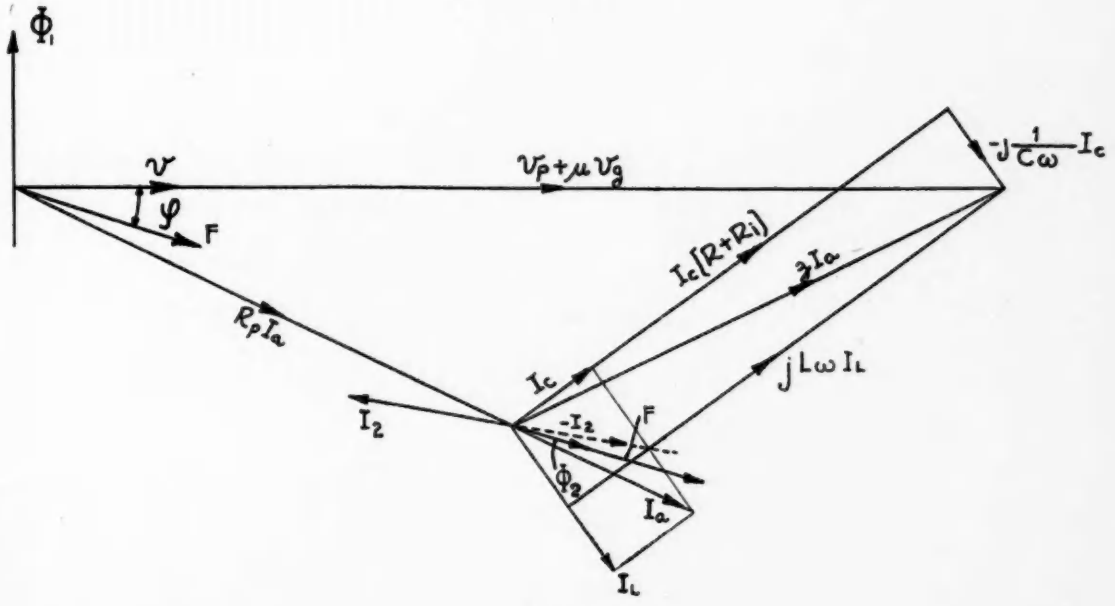


FIG. 29.

For fixed values of E_b and E_c, the auxiliary tube III acts as a purely ohmic resistance R_i if we neglect inter-electrode capacities. Assuming that the impedance takes on a value sufficiently high that the current I_2 will be negligible compared to I_c, Fig. 29 illustrates a vectorial representation of the circuit for $\varphi \neq 0$.

Upon examining this diagram it can easily be seen that the value of φ depends upon R_i. This property can be profitably used for the automatic regulation, by varying R_i with the bias potential E_c and making this potential depend upon φ.

This dependency is obtained by employing a circuit similar to the electronic wattmeter of Turner and McNamara[121] by connecting two auxiliary coils wound upon the same coils as the

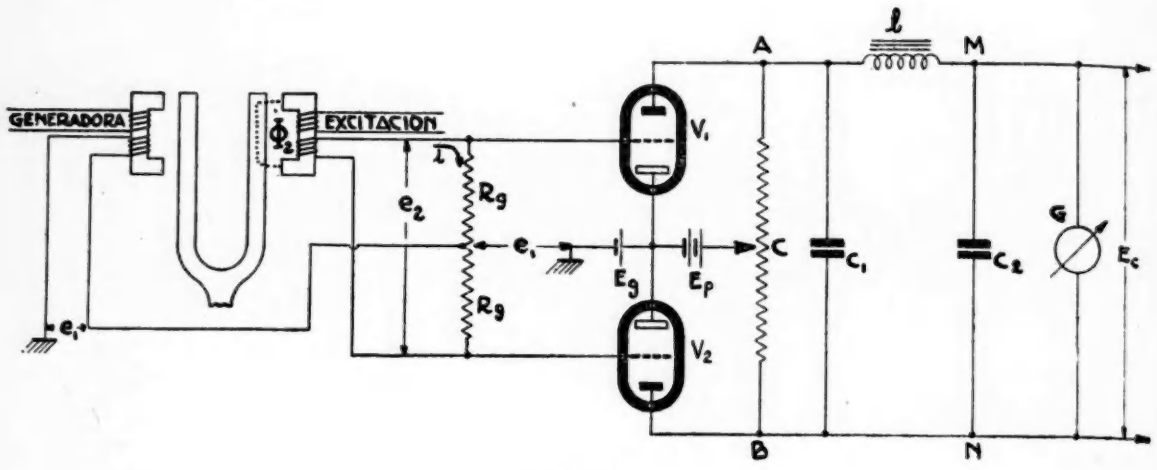


FIG. 30.

excitation and generating coils as shown in Fig. 30.

The potential e_1 is in phase with the velocity and e_2 in quadrature (lagging) with respect to the flux Φ_2, that is to say with the force F, assuming the losses in the coils are negligible and the resistance R_g so high that i will be very small. ($Rg \approx 1$ megohm.)

With this connection, the alternating potential upon the grid of V_1 will be given at any instant by

$$e_1+e_2/2,$$

and that of V_2

$$e_1-e_2/2.$$

If the tubes work upon the parabolic part of the characteristics, the potentials between the points AC and CB will be respectively

$$E_{AC}=k[E+\mu(e_1+e_2/2)]^2,$$

$$E_{BC}=k[E+\mu(e_1-e_2/2)]^2,$$

where $E=E_b-\mu E_c$, μ the amplification factor and k a constant.

The difference in potential between A and B will be:

$$E_{AB}=E_{AC}-E_{BC}=k\left[E\frac{e_2}{2}+\mu e_1\frac{e_2}{2}\right],$$

where e_1 and e_2 are two sinusoidal waves of such a phase that $\xi=\pi/2\pm\varphi$; this expression can be written

$$E_{AB}=k\left[E_1\frac{E_2}{2}\sin\omega t+\mu E_1\frac{E_2}{2}\sin\omega t\sin(\omega t+\xi)\right].$$

Making trigonometric transformations there results:

$$E_{AB}=k\left[E_1\frac{E_2}{2}\sin\omega t - k_2E_1\frac{E_2}{2}\cos(2\omega t+\xi)+k_2E_1\frac{E_2}{2}\cos\xi\right].$$

The filter consisting of condensers C_1 and C_2 and the impedance l eliminates the AC components, giving as a final result:

$$E_c=E_{MN}=k_3E_1E_2\cos\xi=k_3E_1E_2\sin\varphi.$$

In this way the potential applied to the grid of the auxiliary tube III depends upon the magnitude and sign of φ.

In order to effect the automatic control, the potential E_{MN} is connected between the grid of tube III and the cathode, in such a manner that the absolute value of the total negative potential of the grid (E_c+V_{g0}) diminishes when φ acquires negative values. Under these conditions R_1 diminishes and the force F tends to lead with respect to the velocity, automatically decreasing φ as was seen in Fig. 29.

The galvanometer G of Fig. 30 has a zero center scale and gives zero deflection only for the case of $\varphi=0$, thus permitting a continuous observation of the effectiveness of the automatic phase control. The variable potentiometer C allows for compensation for differences between the characteristics of tubes V_1 and V_2 by ad-

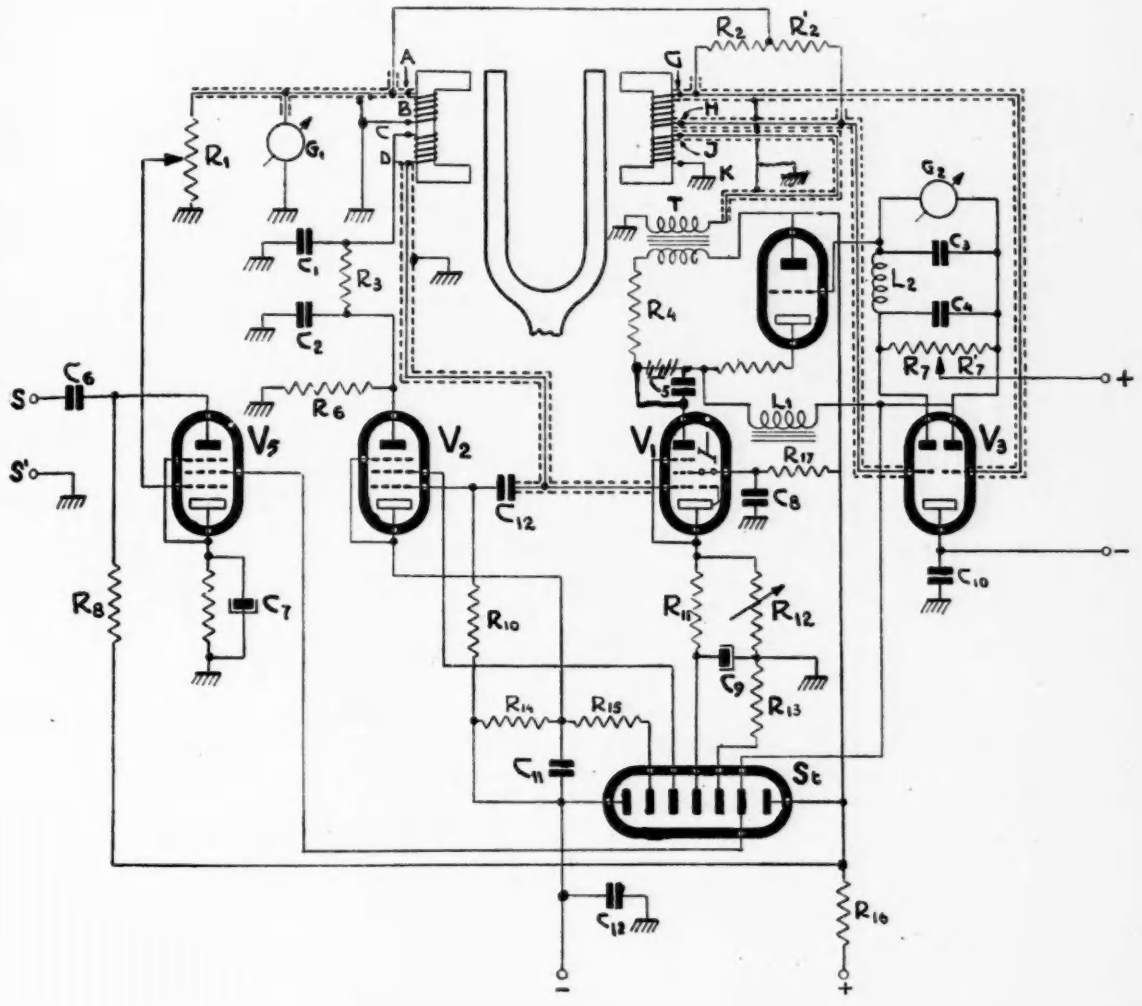


FIG. 31.

justing for zero deflection of the galvanometer in the absence of e_1 and e_2 (tuning fork not operating).

The variable resistance R (Fig. 28) makes possible the variation of the grid operating potential V_{g0} of tube III in order to adjust the internal resistance R_i to the value corresponding to $\varphi=0$ whenever tubes or other circuit elements are replaced.

It is well to point out that the system adopted for producing the regulating voltage, makes the value of this potential independent of the values of e_1 and e_2 for $\varphi=0$ (Eq. (15)). This result is important since e_1 and e_2 vary during operation within the limits of the automatic amplitude control.

EFFECTIVENESS OF THE AUTOMATIC PHASE CONTROL

The double triode Telefunken EDD11[122] was adopted as the control tubes V_1 and V_2 and since $e_1 \approx 1$ volt and $e_2 \approx 2$ volts, the adopted working conditions were [$Ug=0$; $Ua=175$ volts[123]] $E_1=11$ volts; $E_2=22$ volts (formulae (15)). The measurement of E_c was accomplished by the use of a microammeter provided with the proper multiplying resistance so that if 0.2 division is considered the smallest scale reading possible, the instrument can be read to 20 mv.

Under the assumed conditions it is possible to detect a phase difference of 40″.

The measurements obtained for several values of φ combined with the application of formulae

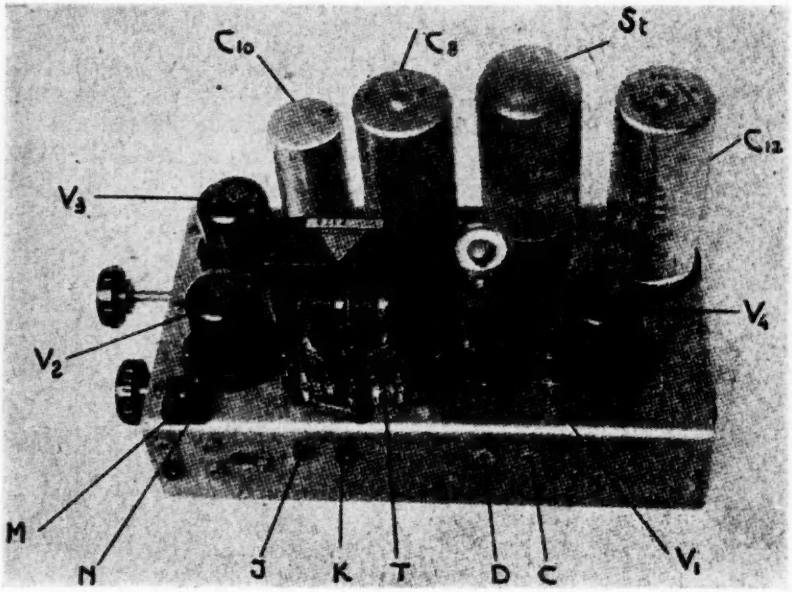


FIG. 32.

(9) gives:

$$\Delta\varphi = 40'', \quad \Delta f \approx 3\times 10^{-10}.$$

Thus the simple observation of G allows us to detect frequency changes caused by changes of phase greater than 3×10^{-10} and when necessary corrections can be made with the manual control R. In most cases this manual control is unnecessary, because of the effect of the automatic control whose effectiveness has been proven by loading the tines of the tuning fork unsymmetrically or by introducing electromagnetic reactions in the core of the excitation coil by the use of an auxiliary short-circuited coil. In all cases the control maintained the phase within the predetermined limits.

EFFECT OF LOAD VARIATIONS ON FREQUENCY

This effect can be eliminated by using a separate tube whose grid is excited with the constant frequency voltage and with the load connected in the plate circuit. The circuit adopted also permits us to obtain constant amplitude which is seen to be important for quantitative measurement purposes.

Figure 31 represents the complete circuit of the tuning-fork generator. A voltage regulator Stabilovolt has been used for the purpose of maintaining the supply voltages constant within certain limits which helps to stabilize the operation of the automatic phase and amplitude regulators.

Figures 32 and 33 (not reproduced herein) represent two views of the apparatus with notations corresponding to those of Fig. 31.

MULTIPLICATION OF FREQUENCY

In order to multiply the frequency, the multivibrator of Abraham and Bloch was adopted. Figure 34 shows the basic scheme of this device.

The theory of the multivibrator[71, 80, 83, 125] indicates that the generated potential e follows a periodic law, with high harmonic content, whose fundamental frequency can be approximately expressed by

$$f_0 = 1/(C_1R_1 + C_2R_2).$$

If the anode potential contains an a.c. component of frequency f_1 near f_0, the frequency of e

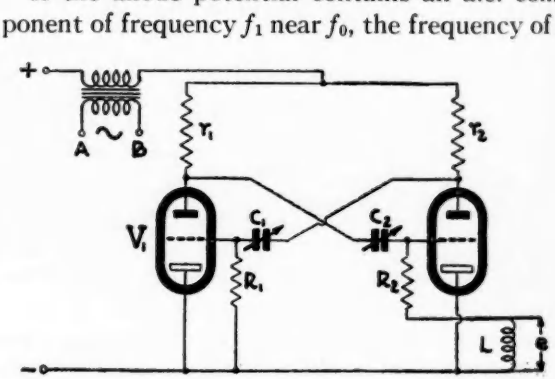


FIG. 34.

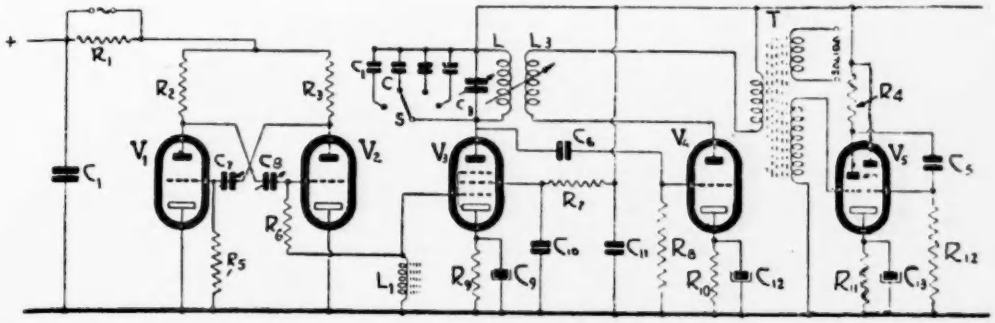


FIG. 35.

will become equal to f_1. Once synchronized, it is possible to vary the constants of the multivibrator within wide limits without losing the control effect.

By amplifying the voltage e with a selective amplifier which permits choosing only one harmonic, it is evidently possible to obtain in the output a voltage of frequency nf_1, whose constancy will depend solely upon the constancy of f_1. This effect has been made use of to obtain a series of discrete frequencies, making use of the previously described tuning fork oscillator as the control element.

The multivibrator and selective amplifier was constructed in a unit following the circuit of Fig. 35.

Figure 36 shows this apparatus, the notations corresponding to those of Fig. 35.

In order to make evident the synchronizing effect of the alternating potential applied in series with the plate supply voltage, simultaneous oscillograms of the synchronizing potential and the output of the multivibrator were made, immediately after adjusting the values of C_1 and C_2 in such a way that the fundamental frequency was approximately 1000 c.p.s. (frequency of the synchronizing potential.)

Figure 37 shows this oscillogram in which can be noted the perfect synchronism obtained.

The selective amplifier has been made regenerative (coils L_1 and L_2) for the purpose of compensating for losses in the coil L_1 of the oscillating circuit. With the coupling adjusted close to the point of oscillation, the circuit becomes extraordinarily selective, rendering it possible to individually select any desired harmonic.

In order to facilitate the adjustment of C_3 to inaudible harmonics, a visual resonance indicator (magic eye) has been placed in the circuit.

The plate circuit of the first amplifier tube consists of separate fixed and variable condensers in combination with several interchangeable inductances which allows the selection of any harmonic from 2 kc to 100 kc.

The condenser C_3 is provided with a scale previously calibrated to indicate the harmonic corresponding to each setting for each coil and parallel fixed condenser combination.

As an example the 14th harmonic is shown oscillographically in Fig. 38 in conjunction with the synchronizing frequency 1000 c.p.s.

In order to obtain frequencies above 100 kc it is necessary to synchronize another multivibrator with the voltage of frequency nf_1 previously multiplied by the first multivibrator, n being a convenient number calculated as before. The reason this double multiplication is necessary is that the harmonics above the 100th as generated by the multivibrator are so attenuated as to be unusable.

With the use of two steps of multiplication we can, without difficulty, reach frequencies as high as 3000 kc for a control frequency of 1000 c.p.s.

RESULTS OBTAINED

In order to verify the constancy of the frequency, measurements were made comparing the multiplied frequency controlled by the tuning fork with the standard frequency distributed by the Physikalisch-Technischen Reichanstalt of Berlin. A cathode-ray tube was used as the indicator by connecting as shown in Fig. 39 (not reproduced herein). The study was made by producing individual variations of each of the

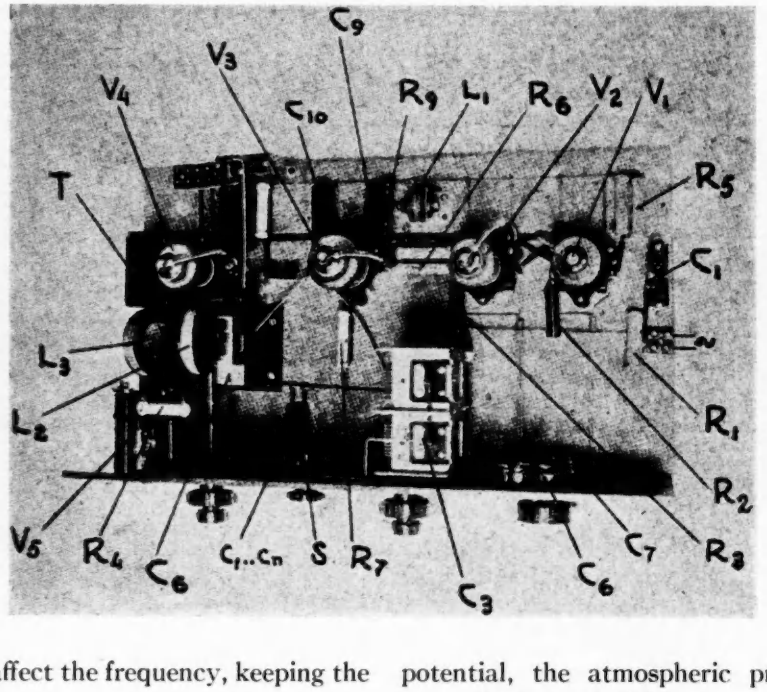


FIG. 36.

variables which affect the frequency, keeping the rest constant.

In normal operation of the apparatus, once the amplitude and phase controls are adjusted and assuming that thermal equilibrium has been reached, the only influences on the frequency are the ambient temperature in which the apparatus is placed (new thermal equilibrium), the supply potential, the atmospheric pressure, and the external magnetic fields.

From the results of the experiments it was found that magnetic fields both continuous and alternating, and variations of atmospheric pressure did not exert a perceptible influence on the frequency produced by the apparatus. This result was to be expected in view of the magnetic

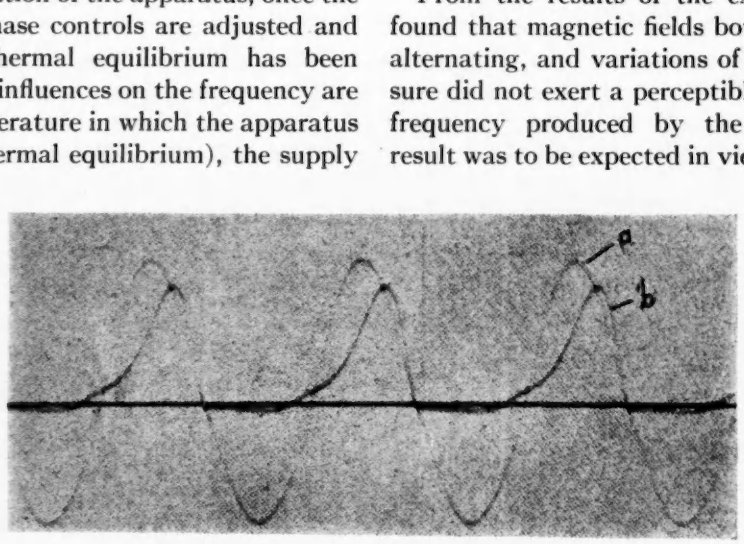


FIG. 37.

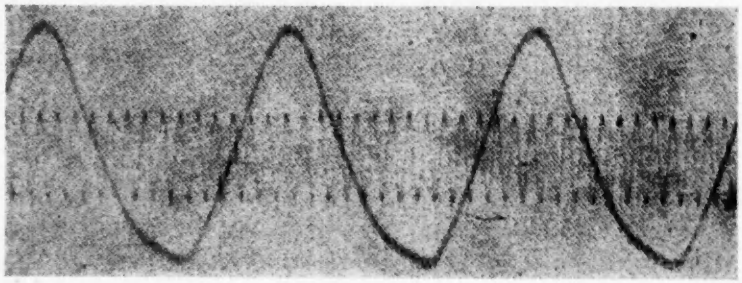


FIG. 38.

shield used and the effect of the automatic phase and amplitude controls. Taking into account that the standard frequency produced by the P.T.R. is constant within one part in 10^8, the study of the frequency variations caused by temperature variations and supply voltages has been carried out by producing variations much larger than one could expect under the worst possible service conditions. The linear dependency of frequency upon these variables was verified and interpolations were made to indicate the maximum variations to be expected under normal operating conditions.

If we are interested not in the absolute value of the frequency but in the frequency variations, the measurements are greatly simplified by connecting the vertical deflecting plates of the oscillograph to the comparison frequency f_x multiplied by 100 and the horizontal plates to the output of the multivibrator controlled by the standard frequency. The chosen harmonic must be such that an ellipse appears upon the screen which, in general, for given small differences in frequency, is of varying dimensions producing the illusion of a rolling hoop. Varying the amplitude of oscillation of the tuning fork one can slow this movement to such an extent that it is imperceptible (equality of frequency). After this is attained it is only necessary to alter the parameter under study (potential or temperature) by noting the change in the rotating movement. The measurement of Δf can be conveniently made by measuring the time for one complete revolution by the use of a chronometer.

In general, a more exact measurement can be made by noting the time necessary for n rotations. The minimum error in measuring the time is obtained by allowing the time period to begin and end at the instant the figure appears as a line on the screen which corresponds to the voltages being in phase (hoop in profile).

Figures 40 and 41 (not reproduced here) represent the set-up of apparatus used for these measurements.

One large series of measurements gave as a result the following mean values:

Variations with Temperature, at Constant Voltage.

$$\left.\begin{array}{l} T_1 = 0^\circ \\ T_2 = 35^\circ \end{array}\right\} \Delta T = 35^\circ; \quad \Delta f = -3.5 \times 10^{-8}.$$

Variations with Voltage, at Constant Temperature.

$$\left.\begin{array}{l} V_1 = 230 \text{ volts} \\ V_2 = 180 \text{ volts} \end{array}\right\} \Delta V = 50 \text{ volts}; \quad \Delta f = -1.7 \times 10^{-8}.$$

Considering that under normal operating conditions, temperature variations of 3.5°C and voltage variations of 5 volts are to be expected and assuming that both produce change of the same sign as the worst possible condition the maximum variation of f that can be expected will be

$$\Delta f = -5.3 \times 10^{-9}.$$

It has been noticed that when first turning on the apparatus as when causing a sharp change of external temperature, it is to be expected that a time of 3 to 5 hours will be needed before reaching the new point of thermal equilibrium. During this time the frequency varies continuously, approaching the final value f_2 asymptotically.

ABSOLUTE MEASUREMENT OF FREQUENCY

Once the constancy of the frequency is demonstrated an absolute measurement can be made by operating a synchronous clock from the output frequency of the tuning fork. The clock is constructed mechanically so that it will indicate correct sidereal time when supplied with exactly 1000 c.p.s. The absolute measurement of frequency is thus reduced to the determination of the correction of the clock by observation of the stars by current astronomical methods.

CONCLUSION

In the foregoing a standard generator has been described which unites to a high degree all of the necessary requisites of such instruments.

The author wishes to express his appreciation to the director of AEG-Berlin, Engineer R. Reese and to the personnel of the Research Laboratory of this company, especially to the Chief Engineer, Czygan, for the effective collaboration he furnished which made possible the accomplishment of this work in spite of the natural difficulties created by the war.

BIBLIOGRAPHY

1. F. Vilbig, *Lehrbuch der Hochfrequenztechnik* (1939), p. 516.

2. H. Reich, *Theory and applications of electron tubes* (1939), p. 336.

3. S. L. Freimann, "Die angenäherte theorie des magnestostrikitiven generators," Jb. d. d. T. u. T. [6] **34**, 219 (1929).

4. G. W. Pierce, "Magnetostriction oscillators," Proc. Am. Acad. Sci. **63**, 1 (1928).

5. G. W. Pierce, "Magnetostriction oscillators," Proc. I. R. E. **17**, 42 (1929).

6. J. H. Vicent, "Experiments on magnetostrictive oscillators at radiofrequencies," Proc. Phys. Soc. **41**, 476 (1929).

7. I. Simon, "Über die Einsenkurve eines magnetostriktiven Resonators," Hoch:tech. u. Elek:akus. **50**, 54 (1937).

8. G. W. Pierce and A. Noyes, "An improved magnetostriction oscillator," J. Acous. Soc. Am. **9**, 185 (1938).

9. J. Ide, "Medidas sobre el vibrador a magnetostricción," Proc. I. R. E. **19**, 1216 (1931).

10. J. Ide, "Oscilador a magnetostricción con bajo coeficiente de temperatura de la frecuencia," Proc. I. R. E. **22**, 177 (1934).

11. F. Vilbig, *Lehrbuch der Hochfrequenztechnick* (1939), p. 510.

12. D. W. Dye, "The piezo-electric quartz resonator and its equivalent electrical circuit," Proc. Phys. Soc. **38**, 399 (1926).

13. K. Heegner, "Über messungen an piezo-elektrischen Kristallen," Telefunken-Ztg. **8**, No. 44, 60 (1926).

14. A. Meissner, "Über piezo-elektrische Kristalle bei Hochfrequenz," E. N. T. **11**, 401 (1926).

15. A. Scheibe, "Piezoelektrische. Resonanzerscheinungen," Jb. d. d. T. u. T. [1] **28**, 15 (1926).

16. Fr. Seidl, "Der Selbstönende Kristall," Physik. Zeits. **27**, 64 (1926).

17. Fr. Seidl, "Neue Beobachtungen am Selbstönenden Kristall," Physik. Zeits. **27**, 816 (1926).

18. A. Crossley, "Piezo-electric crystal-controlled transmitters," Proc. I. R. E. **15**, 9 (1927).

19. E. Giebe and A. Scheibe, "Piezoelektrische Erregung elastischer Schwingungen," Zeits. f. Instrumentenk. [6] **47**, 273 (1927).

20. E. Giebe and A. Scheibe, "Piezoelektrische Erregung von Dehnungs-Biegungs- und Drillungsschwingungen bei Quarstäben," Zeits. f. Physik **46**, 607 (1927).

21. H. Heegner, "Über Messungen an piezoelektrischen Kristallen," Jb. d. d. T. u. T. [6] **29**, 177 (1927).

22. A. Hund, "Note on piezoelectric generators with small back action," Proc. I. R. E. **15**, 725 (1927).

23. A. Meissner, "Über piezoelektrische Kristalle bei Hochfrequenz," Jb. d. d. T. u. T. [1] **29**, 20 (1927).

24. K. S. Van Dyke, "The piezo-electric resonator and its equivalent network," Proc. I. R. E. **16**, 742 (1928).

25. F. W. Hehlganz, "Über piezoquartz-platten als Sender und Empfänger hochfrequenter akustischer Schwingungen," Ann. d. Physik **86**, 587 (1928).

26. F. Gerth and H. Rochow, "Die Temperaturabhängigkeit der Frequenz des Quarzresonators," E. N. T. [12] **5**, 549 (1928).

27. E. Giebe and A. Scheibe, "Piezoelektrische Kristalle als Frequenznormale," E. N. T. [2] **5**, 65 (1928).

28. A. Hund, "Notes on quartz plates, air gap effect and audio frequency generation," Proc. I. R. E. **16**, 1072 (1928).

29. R. S. Strout, "The temperature coefficient of quartz crystal oscillators," Phys. Rev. **32**, 829 (1928).

30. Earle M. Terry, "The dependence of the frequency of quartz piezo electric oscillators upon circuit constants," Proc. I. R. E. **16**, 1486 (1928).

31. Yasusi Watanabe, "Der piezoelektrische Resonator in Hochfrequenz-schwingungskreisen," E. N. T. [2] **5**, 45 (1928).

32. L. P. Wheeler and W. E. Brown, "A new type of standard frequency piezoelectric oscillator," Proc. I. R. E. **16**, 1035 (1928).

33. F. R. Lack, "Observations on modes of vibration and temperature coefficients of quartz crystal plates," Proc. I. R. E. **17**, 1123 (1929).

34. Shogo Namba and Sadao Matsumura, "General properties of a piezo-electric quartz and the value of a quartz oscillator as a frequency standard," Res. Elec. Lab. Tokio No. 248 (1929).

35. O. Tope, "Über den Schwingkristall," Physik. Zeits. **30**, 585 (1929).

36. E. Giebe and A. Scheibe, "Transversalschwingende Leuchtresonatoren als Frequenznormale in Bereich von 1000–20,000 Hz.," Jb. d. d. T. u. T. [5] **35**, 165 (1930).

37. J. R. Harrison, "Pushpull piezo-electric oscillator circuits," Proc. I. R. E. **18**, 95 (1930).

38. A. Hund and R. E. Wright, "New piezo oscillations with quartz cylinders cut along the optical axis," Bur. Stand. J. Res. **4**, 383 (1930).

39. H. J. Lucas, "Some developments of the piezo-electric crystal as a frequency standard," J. Inst. Eng. **68**, 855 (1930).

40. J. E. P. Vigoureux, "The valve-maintained quartz oscillator," J. Inst. Elec. Eng. **68**, 265 (1930).

41. H. Straubel, "Schwingungsform und Temperaturkoeffizient von Quarts-oscillatoren," Jb. d. d. T. u. T. [1] **38**, 14 (1931).

42. E. Habann, "Der Schwingkristall und seine technische Gestaltung," Physik. Zeits. **33**, 615 (1932).

43. A. Honot, "Le quartz piézo-électrique. Ses applications á la télégraphie et la téléphonie sans fil," Rev. Gen. Elec. **31**, No. 6, 177 (1932).

44. F. Kiebitz, "Die Stabilisierung der Hochfrequenzgeneratoren," T. F. T. [9] **21**, 235 (1932).

45. T. D. Parkin, "The Quartz Oscillator," Marconi Rev. **37**, 1 (1932).

46. E. Schwartz, "Experimentelle Untersuchungen über die piezoelektrischen und dielektrischen Eigenschaften des Seignettesalzers," E. N. T. [12] **9**, 481 (1932).

47. E. Giebe and A. Scheibe, "Über Leuchtresonatoren als Hochfrequenznormale," Hoch:tech. u. Elek:akus. **41**, 83 (1933).

48. R. Bechmann, "Entwicklung der Quarzsteuerung der Telefunken Grossender," Telefunken Ztg. **15**, No. 68, 16 (1934).

49. R. Bachmann, "Über die Temperaturkoeffizienten der Eigenschwingungen piezoelektrischer Quarzplatten und Stabe," Hoch:tech. u. Elek:akus. **44**, No. 5, 145 (1934).

50. I. Koga, "Thermal characteristics of piezoelectric

oscillating quartz plates," Rep. Radio Res. Japan **4**, 61 (1934).

51. S. Matsumura and S. Kanzaki, "Quartz plates with a very small temperature coefficient of oscillation frequency," Rep. Radio Res. Japan **4**, No. 2, 105 (1934).

52. H. Mögel, "Ein Betriebswellenmesser mit Anschluss an die Quarzuhren der Physikalish Technischen Reichsansalt," T. F. T. **23**, No. 6, 148 (1934).

53. A. Scheibe and U. Adelsberger, "Die technischen Einrichtungen der Quarzuhren der Physikalisch-Technischen Reichsanstalt," Hoch:tech. u. Elek:akus. **44**, No. 4, 118 (1934).

54. U. Adelsberger, "Zeit und Frequenzmessung hoher Genauigkeit," E. N. T. **12**, No. 3, 83 (1935).

55. U. Grützmacher, "Piezoelektrischer Kristall mit sehr tiefer Eigenfrequenz," E. N. T. **12**, No. 8, 257 (1935).

56. W. F. Diehl, "A new piezoelectric quartz crystal holder with thermal compensator," R. C. A. Rev. **1**, No. 2, 86 (1936).

57. R. O. Schuhmacher, "Untersuchungen an querschwingenden Quarzplatten," Telefunken Ztg. **18**, No. 76, 16 (1937).

58. N. C. Stamford, "The production of Rochelle salt piezoelectric resonators having a pure longitudinal mode of vibration," Proc. I. R. E. **25**, No. 4, 465 (1937).

59. R. Bechmann, "Über Schaltungen für piezoelektrische Quartzoszillatoren und Resonatoren zur Frequenztabilisierung und als Selektionsmittel," Telefunken Ztg. **19**, No. 78, 60 (1938).

60. K. Iower and M. Harvey, "A crystal oscillator and frequency multiplifier," Television London **11**, No. 122, 235 (1938).

61. I. Koga and Shoyama, "Transient frequency variation of crystal oscillators," Electrotech. J. Tokyo **2**, No. 9, 199 (1938).

62. E. C. Metschi, "Wesen und anwendung der Piezoelektrisität," Elektrotech. Zeits. **59**, No. 31, 819 (1938).

63. A. Scheibe, "Genaue Zeitmessung," Ergeb. d. exakt Naturwiss. **15**, 262 (1936).

64. A. Scheibe, "Normal frequenzen der Phys.-Technisch. Reichsanstalt," Hoch:tech. u. Elek:akus. **53**, 145 (1939).

65. A. Scheibe and U. Adelsberger, Physik. Zeits. **40**, 216 (1939).

66. G. Wuckel, "Biegequartz-Generators. Europäischer Ferns," **53**, 267 (1939).

67. R. Bechmann, "Dickensschwingungen piezoelektrisch erregter Kristallplatten," Hoch:tech. u. Elek:akus. **56**, 14 (1940).

68. E. Giebe and Blechschmidt, "Über Drillungsschwingungen von Quarzstäben und ihre Benutzung für Frequenznormale," Hoch:tech. u. Elek:akus. **56**, 65 (1940).

69. L. Rohde and R. Leonhardt, "Quarzuhr und Normalfrequenzgenerator," E. N. T. **17**, 117 (1940).

70. J. Zenneck, "Die Transformation der Frequenz," Jb. d. d. T. u. T. **7**, 412 (1913).

71. H. Abraham and Bloch, "Multivibrator," Ann. d. Physik **12**, 237 (1919).

72. O. Emersleben, "Theorie der Frequenzvielfachung mittels Oberschwingungen und durch Stosserregung," Jb. d. d. T. u. T. **24**, 105 (1934).

73. J. K. Clapp, J. Opt. Soc. Am. **15**, 25 (1927).

74. J. Koga, "A new frequency transformator or frequency changer," Proc. I. R. E. **15**, 669 (1927).

75. W. Morrison, Proc. I. R. E. **17**, 1103 (1929).

76. W. Bunimowitsch, "Über Frequenzverdopplung mit Elektronenrohren," Jb. d. d. T. u. T. **35**, 223 (1920).

77. Mercier, "Sur la syncronisation harmonique des oscillateurs electriques," Comptes rendus **174**, 448 (1922).

78. V. Wologdin, "Frequenzvervielfachung durch Anwendung eines Kondensators mit Seignettesalzdielektrikum," Zeits. f. tech. Physik **13**, 82 (1932).

79. R. Golicke, "Teilung und Vervielfachung von Frequenzen," E. N. T. **15**, 134 (1938).

80. Reich, *Theory and App. of Electron Tubes*, "Use of Relaxation Oscillators in Frequency Transformation" (1939), p. 359.

81. L. M. Hull, "Un método conveniente para referir patrones secundarios de frecuencia a patrones de intervalo de tiempo," Proc. I. R. E. **17**, 552 (1929).

82. Horton and Morrison, "Determinación de frecuencia de precisión," Proc. I. R. E. **16**, 137 (1928).

83. Y. Namba, "The establishment of the Japanese radiofrequency standard," Proc. I. R. E. **18**, 1017 (1930).

84. A. G. Rossi, "Diapason electromagnetico," Il nuovo cimento **16**, 97 (1908).

85. Eecles, Phys. Soc. London **31**, 269 (1939).

86. S. Butterworth, "El mantenimiento de un sistema vibrante por medio de un triodo," Phys. Soc. London **32**, 345 (1920).

87. W. Cady, "Guía piezoeléctrica de diapasones y barras," Phys. Rev. **21**, 371 (1923).

88. T. G. Hodgkinson, "Diapasones mantenidos a válvula sin condensadores," Phys. Soc. London **38**, 24 (1926).

89. D. Dye, "El diapasón mantenido a válvula como un patrón de tiempo de precisión," Proc. Roy. Soc. London **103**, 241 (1923).

90. F. Robin, "Sur la hauteur du son dans les alliages et ses variations en fonction de la temperature," Comptes rendus **153**, 665 (1911).

91. Mercadier, Comptes rendus **83** (1876).

92. Kaiser, Wied Ann. **8** (1879).

93. Konig, Wied Ann. **9** (1880).

94. Michelson, Ann. Inst. Sci. **25** (1883).

95. Pierpaoli, "Variación de la altura de un diapasón producida por magnetización," Att. Acad. Lincei **3**, 368 (1894).

96. Woodruff, Phys. Rev. **16** (1903).

97. Zambiasi, "Verificación del diapasón normal del Ufficio Centrale Italiano," Att. Acad. Lincei **6**, 34 (1906).

98. Tuma, "Sobre la observación del batimento de dos diapasones con la ayuda del micrófono," Sitz. d. Math. Naturwiss. Kl. d. Kaiserlichen Akad. d. Wiss. Wien **98**, 1028 (1899).

99. Roman and Dey, "Un nuevo método para la determinación absoluta de la frecuencia," Proc. Roy. Soc. London **95**, 533 (1919).

100. F. Robin, "Variación de la altura and de la duración del sonido en los aceros en función de la temperatura. Diapason invariable," J. de Phys. **2**, 298 (1912).

101. *Handbuch der Physik*, Vol. VIII Akustik (1927).

102. Chevenard, Travaux et mémores du Bureau International des Poids et Mesures **17**, 1–142 (1927).

103. Guillaume, Revue de Metallurgie **25**, 35 (1928).

104. Fahlenbrach and Meyer, "Werkstoffe mit besonderer Temperaturabhängigkeit des Elastizitätsmodulus," Zeits. f. tech. Physik **2**, 40 (1940).

105. W. A. Marrison, "Diseño de termostatos para patrones de frecuencia," Proc. I. R. E. **16**, 976 (1928).

106. V. Heaton and Brattain, Diseno de un oscilador piezoeléctrico portátil con control de temperatura **18**, 1239 (1930).

107. J. K. Clapp, "Control de temperatura para patrones de frecuencia," Proc. I. R. E. **18**, 2003 (1930).

108. Landolf and Boernstein, *Phys. Chem. Tabellen.*

109. Kohlrausch, *Praktische Physik.*

110. Groeber and Poensgen, *Wärmeiabellen* (Holborn, Scheel, and Henning).

111. Mesny, *Radioélectricité Generale*, Vol. 1, p. 160.

112. F. Kolacek, "Über den Einfluss des den Schall leitenden Medius auf in ihm Schwingende Tonquellen," Ann. d. Physik. Chemie **7**, 23 (1879).

113. Dayton C. Miller, "Influence of amplitude of electromagnetic driving on the frequency of tuning forks," Phys. Rev. **11**, 497 (1918).

114. Horton, "Vacuum-tube oscillators," Bell Sys. Tech. J. **3**, 508, 524 (1924).

115. Thomas, *Theory and design of valve oscillators* (Chapman and Hall Ltd., London, 1939), p. 10.

116. J. E. Scheel, "Die Niederfrequenz-Regelröhre mit Abstimmanzeiger. EFM 11," *Die Telefunken Röhre*, Volumen extroardinario agregado al volumen **13**, p. 72 (1938).

117. Van der Pol, "The triode as a generator of oscillations," Rad. Rev. **1**, 701–754 (1920).

118. Lienard, "Estudio de oscilaciones entretenidas," Rev. Gen. d'Elec. **23**, 801–946 (1928).

119. Usui, "Operating mechanisms of negative-resistance oscillators," J. I. E. E. Japan **51**, 64–115 (1931).

120. Usui, "Analytical studies on the fundamental characteristics of triode oscillators," J. I. E. E. Japan **52**, 44 (1932).

121. Turner and McNamara, "Un. Wattimetro a tubo electrónico un voltímetro y un puente cambiador de fase," Proc. I. R. E. **18**, 1743 (1930).

122. J. E. Scheel, "Die Endstüfe für den Kraftwagenempfäger mit den Röhren EBC11 und EDD11," *Die Telefunken Röhre*, Vol. Extraordinario agregado al volumen **13**, p. 113 (1938).

123. L. Ratheiser, "Telefunken Rundfunkröhren," *Eigenschaften und Anwendung*, Tercera edicion (1938), p. 263.

124. Schräter and Körös, "Der Stabilisator und seine Anwendungen," Die Telefunkenröhre **4**, 164 (1935).

125. D. W. Dye, "A self contained standard harmonic wave meter," Phil. Trans. **224**, 259 (1924).

Investigation of Resonances in Filaments by Mechanical Actuation and Measurement of Electromagnetically Induced Voltages*

R. W. CARLISLE AND H. W. KOREN
Sonotone Corporation, Elmsford, New York
(Received May 26, 1945)

INTRODUCTION

THE writers have developed the following method of testing undesired vacuum-tube filament vibrations, commonly known as microphonics. The instrument gives an indication of the movement of the filament in a magnetic field.

Considered broadly, the method consists of vibrating a vacuum-tube "mount" by a suitable vibrator, actuated by an oscillator, meanwhile immersing the mount in a magnetic field of B gauss. This is shown in Fig. 1. The voltage generated in the moving conductor, as well known, is

$e = B\,lv$ abvolts, where
l = length of conductor in cm and
v = velocity in cm per second

This voltage is amplified in an electronic amplifier and detected with headphones and an a.c. meter as indicated.

STRUCTURE OF TEST JIG

The practical construction of a jig constructed by the writers is shown in Fig. 2. The V-shaped magnet system is mounted on top of the iron shield, and the vibrator on the bottom, the shield serving as mechanical support as well as shielding the tube mount against a.c. fields from the vibrator. The vibrator consists of a high-grade, well-damped phonograph pick-up made by RCA. Vibrations are transmitted by a channel-shaped aluminum drive rod, fastened to a dummy phonograph needle at the lower end and supported in a drive rod guide near the top. A clearance hole for the drive rod is provided in the iron shield and magnetic system. The tube mount (or a complete tube) is touched to the upper end of the drive rod. Undesirable rubbing against the pole pieces is prevented by lining them with soft Vinylite. A socket is mounted on a hinged bracket, which is swung up to insert a tube base and swung down to immerse the mount in the field.

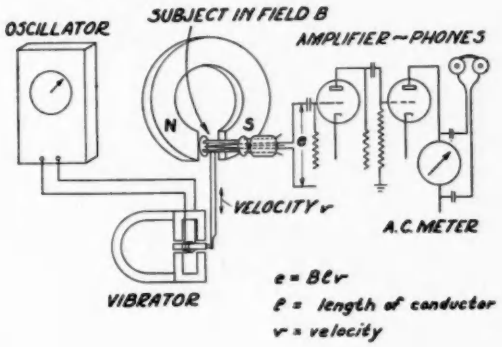


FIG. 1.

ILLUSTRATIVE RESULTS

A sample calculation will now be mentioned to show the amplitudes being measured. In the particular jig described the magnetic field strength is 600 gauss (in an air gap one-half inch long). For a generated voltage of one-tenth of a millivolt, a filament of which 2 cm are immersed in the field has a velocity of $E10^8/B1$ or 8.3 cm/sec. This corresponds to an amplitude of velocity/$2\pi \cdot$freq. or .0013 cm at 1000 cycles.

The parts of a tube mount which are essential to a consideration of microphonics are shown on the lower right-hand side of Fig. 3. The filament strands are supported by tabs in the bottom mica and by a tensioning spring extending through the upper mica. This spring is welded to a support

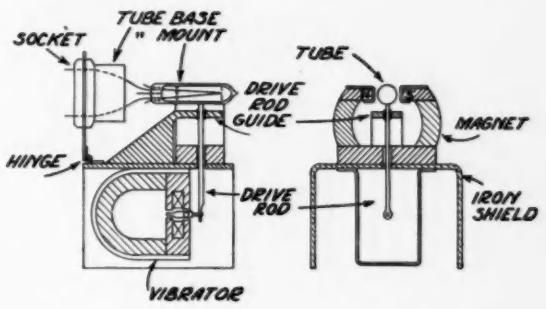


FIG. 2.

* Presented at the New York meeting of the Acoustical Society, May 11, 1945.

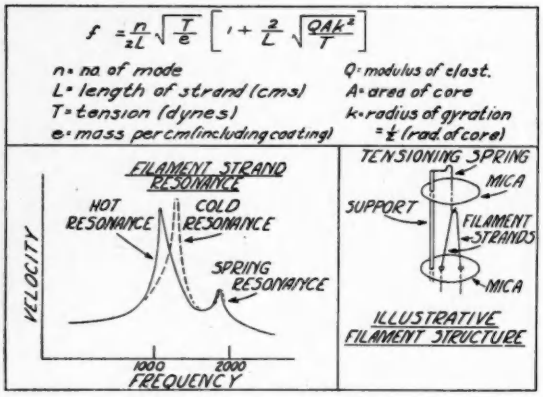


FIG. 3.

wire, which is generally a part of a suppressor grid.

When this structure is actuated by vibrations of audiofrequency, the voltage generated in the filament has resonances, as typified by the curves at the left of Fig. 3.

The most radical contribution which this jig makes to vacuum-tube technique is that vibrations may be investigated with filaments either hot or cold, in open mounts or in sealed bulbs.

The filament expands when heated, thus relaxing its tension, so that the resonance frequency of the strands when hot is lower than when cold. Resonances of the two sides of a *V* filament are often slightly different in frequency due to unequal tension, but are here shown identical for simplicity. Standing waves occur in the tensioning spring; this spring is not changed in temperature nor much in stiffness by heating the filament, hence the frequency of such resonances is essentially constant, as indicated. Resonances in the spring can be distinguished from those in the filament by means of this constancy. The validity of these observations was checked by microscopic observation of the vibrations.

One purpose of the jig is to control filament tension, since excessive tension shortens filament life. The frequency of resonance of a filament is shown in the formula at the top of Fig. 3, which is adapted from Morse.[1] The meaning of each letter is indicated in the chart. The first term is a function of tension alone, while the second utilizes the stiffness of the filament considered as a beam. The relative magnitudes of the two terms may be quickly checked by noting the resonance frequency of a given filament when completely relaxed, which is apt to occur at say 600 cycles, and comparing it to a normally tensioned filament (approximately 1200 cycles).

By controlling the filament tension as a function of its resonance frequency, tension stresses may be kept within any desired limits.

The rate of decay is important in gauging the objectionable character of microphonics, and can be directly observed with earphones.

[1] P. M. Morse, *Vibration and Sound* (McGraw-Hill Book Company, Inc., New York, 1936), pp. 63, 112, and 131.

Group Audiometry

J. DONALD HARRIS
Medical Research Laboratory, Submarine Base, New London, Connecticut
(Received March 1, 1945)

The use of commonly available apparatus is described for a group test of auditory acuity as a function of frequency. Reliability is only slightly less than that of a careful individual examination. The techniques which produce highest reliability are described. Validity is satisfactory in terms of deviation from results of an individual test. The test is simple to take. Several checks on malingering are provided which make group audiometry practicable with populations not highly selected for age or intelligence.

INTRODUCTION

IN order to handle more efficiently the taking of pure-tone audiograms, the standardization of a group test which could replace the individual examination was attempted. This paper presents data on one such test which has been found satisfactory.

The present work has modified the procedures of the Bell Telephone engineers, notably Munson,[1] and Steinberg, *et al.*,[2] in the direction of efficiency and reliability, and has established the applicability of group audiometry in a routine military situation.

APPARATUS

Two sources of sound have been used, a beat-frequency oscillator with provision for intensity control, in 5 db steps of frequencies up to 3000 c.p.s., and a Western Electric 6B audiometer. These have been coupled through potentiometers respectively to 12 Baldwin magnetic resonant phones, and to 12 Permoflux ANB-H-1A dynamic non-resonant phones. For any one headband, both the phone and a wooden dummy on the other side were fitted with Harvard doughnut cushions Type 6B. Although the first apparatus can be depended upon, the 6B audiometer and the ANB-H-1A phones are to be preferred since results from this equipment are somewhat more reliable, the frequency range is extended several octaves, and the necessity of correcting for phones unequal in response is obviated. This paper presents data largely from the latter equipment.

All tests were conducted in a room which for all practical purposes may be called completely soundproof.

The use of the ANB-H-1A phones was necessary since we could not obtain a number of Western Electric 705A phones to connect in series-parallel so that the gang impedance would match that of the single, standard phone. The ANB-H-1A phones connected in parallel are of somewhat different impedance from the single 705A receiver, and necessitated recalibrating the audiometer as follows:

a. A subject with good ears was given several individual audiograms with the 705A phone, and reliable means were obtained for the octaves 256–8192 c.p.s.

b. The same subject was given an audiogram using each phone of the group audiometer, indicating his responses by a flash of light just as usual with the 705A.

c. The mean threshold setting of the 12 phones for any one frequency was then computed and (corrected for any deviation of the subject's ear from "normal" on the 705A) used as the zero point for that frequency on the group audiometer. Now, when the intensity control knob is set at the new zero, or "normal," position for that frequency, a tone of normal threshold intensity may be heard at each group phone.

The recalibration was completed by this procedure in terms of the original calibration of the 6B instrument. Even though some range is lost, tones sufficiently loud for all practical purposes may be presented. In terms of db above threshold, the intensities as shown in Table I are maximum for our equipment.

TABLE I.

256 c.p.s.	−70	2048 c.p.s.	−99
512	−81	4096	−91
1024	−78	8192	−80

[1] W. A. Munson, Unpublished Research Memorandum, Case 20871-2. Bell Telephone Laboratories, Inc., New York (1937).

[2] J. C. Steinberg, H. C. Montgomery, and M. B. Gardner, J. Acous. Soc. Am. 12, 291–301 (1940).

Interphone comparisons were made by requiring a group of 12 men to sit in each seat in succession and take an audiogram for each ear with each phone. The mean thresholds for all phones can be directly compared since the same ears enter into each mean. We found that for the first 12 phones we installed, out of 396 interphone comparisons only three showed significant differences. The actual means are shown in Table II and may be taken as representative of the extent to which correction of individual phones will be found necessary. So few corrections as we found necessary may be made with little loss of time, or it would usually be found possible to discard one or more atypical phones and find more closely comparable ones.

PROCEDURE

Each subject is provided with headphone, pencil, and answer blank. A tone of the desired frequency at an intensity 30 db above threshold is presented in either 1, 2, or 3 short spurts, each spurt about $\frac{1}{2}$ second duration, separated by about $\frac{1}{4}$ second intervals. The subject is required, on one answer form, to indicate the presence or absence of a tone or, on a different answer blank used at other times, to indicate *how many* spurts of tone he heard (the "pulsing tone" method of Munson). Then in succeeding steps of 5 db a descending series of intensities to 10 db below normal threshold is presented, and without pause an ascending series follows, repeating all the steps given in the descending. An occasional item is omitted. A subject's limen is taken as that lowest intensity setting marked correctly at least 50 percent of the time, provided all louder levels are marked correctly. Those subjects who do not hear at the 30 db level are screened out and put through exactly the same procedure except that the intensity steps range from 60 to 30 instead of from 30 to −10 db.

To describe the reliability of the procedure, the mean deviation of retest scores from test scores is used. This statistic has the advantage over the reliability coefficient and the standard error of estimate in that, respectively, the range of ability does not exert an undue influence, and it becomes possible to apply the usual tests for significance between alternative procedures.

TABLE II. Comparison of output of Permoflux Phones. Entry: Mean hearing loss in decibels (same 24 ears at all phones).

Phone	256	512	1024	2048	4096	8192
1:	5.55	2.50	−2.25	3.85	2.85	9.35
2:	7.25	4.15	−1.30	5.60	4.40	10.95
3:	7.25	3.85	−1.95	6.65	2.85	11.15
4:	6.15	3.05	−2.75	5.00	2.50	12.80
5:	4.40	2.50	−1.70	6.15	4.40	13.10
6:	6.95	2.25	−2.50	3.60	3.60	11.25
7:	5.00	.85	−2.50	7.75	1.35	4.65
8:	10.55	5.85	.60	6.40	2.85	14.05
9:	5.10	2.25	−1.95	6.70	3.25	9.70
10:	8.05	4.15	− .55	4.40	3.25	13.75
11:	7.15	4.70	−1.35	8.25	4.40	10.95
12:	7.40	4.15	−1.10	6.30	4.35	16.30
Range of S.E.:	1.00–1.32	.87–1.30	1.08–1.62	1.36–2.19	1.11–1.76	2.07–3.11

Note: Of 396 interphone comparisons, 3 show significant differences; accordingly we credit an audiogram from Phone 8, 5 db at 256; from Phone 7, discredit 5 db at 8192; and from Phone 12, credit 5 db at 8192.

TABLE III. Test-retest audiometer reliability. Entry: Mean deviation, test-retest, ±1 standard error.

	1 Mean deviation	2 Mean deviation	3 Mean deviation	4 Mean deviation	1 vs. 4 Difference	1 vs. 4 Critical ratio
c.p.s.						
256	2.15 .35	2.65 .28	3.05 .26	3.80 .35	1.65	3.3
512	2.10 .37	1.95 .19	2.35 .23	3.35 .33	1.25	2.5
1024	2.55 .38	1.45 .21	2.00 .20	2.05 .35	.50	.9
2048	2.15 .35	2.30 .27	3.50 .33	3.85 .43	1.70	3.1
4096	4.10 .43	2.90 .36	3.65 .38	5.10 .59	1.00	1.3
8192	3.35 .40	2.65 .33	4.85 .39	5.40 .60	2.05	2.8

Legend: 1—Individual examination (N =64).
2—Group test, phones not moved (N =120).
3—Group test, test-retest phones equivalent in output (N =150).
4—Group test, test-retest phones at random (N =72).

RESULTS

Table III presents test-retest audiometer reliability under four conditions, first an individual examination, second a group test in which the phones were not moved between test and retest, third a group test in which the retest phone differed very little from the test phone according to Table II, and fourth a group test in which the retest phones were selected at random. A general trend toward lowered reliability is seen from condition 1 to 4 (though this is by no means

TABLE IV. Effect of doughnut cushions on reliability. Entry: Difference between mean deviations of two procedures. One in which phones were not moved between test and retest, and one in which they were moved; and critical ratio of the difference.

	Baldwin		ANB–H–1A	
	Difference	Critical ratio	Difference	Critical ratio
256	1.56	3.1	.40	1.0
512	1.90	3.8	.40	1.3
1024	1.80	3.5	.55	1.8
2048	1.10	2.1	1.20	2.8

always clear). The difference between reliabilities of the individual examination and the group test with equivalent phones (condition 3) is never more than 1.5 db and not always in favor of the individual examination. It can be concluded from this table that the group procedure *per se* is as reliable as the usual clinical examination.

When the group retest phone is chosen at random (condition 4), however, the significance of the difference between individual and group procedures is increased. The last columns of Table III compare the two and give the critical ratios. Evidently changing phones for a group retest lowers reliability by a small but generally significant amount.

The ANB-H-1A phones are more reliable on test-retest than the Baldwin. Whether the added reliability is attributable in part to the doughnut cushions encasing the ANB-H-1A phones, is discovered by comparing the two phones, (a) when the phone is not moved or touched between test and retest, and (b) when the phone is removed and replaced for the retest. There will of course be an increase in variability of condition (b) over (a); if this increase in variability is greater for the Baldwin phones than for the ANB-H-1A, the doughnut cushions would seem to make for less variability, that is, greater reliability.

Forty-eight men were tested with Baldwin (cushionless) phones under both conditions (a) and (b); the increase in variability of (b) over (a) is Column 1 of Table IV. Column 3, on the other hand, is the increase in variability of (b) over (a) when the same subjects were given those conditions with the ANB-H-1A (doughnut) phones.

Table IV shows, for the two types, the increase in mean deviation which we may expect from changing phones for retest. The ANB-H-1A phones show less variability, and it is clear that at least for low frequencies the doughnuts are a source of added reliability. The possibility that the air enclosed by them might reinforce certain frequencies, while at other phone positions on the head other frequencies might be affected, does not seem to be true in the octave audiogram.

The effect of the answer form on reliability may be summarized by saying that it makes little difference whether the subject simply reports the presence or absence of a tone or whether he is required to report how many spurts he hears in a certain time-interval. Critical ratios between the two procedures are entirely insignificant, and no trend in favor of either may be found. Nevertheless the latter procedure is to be preferred in general since it contains an additional check on malingering. In a fair percentage of those papers in which the descending and ascending series yield different thresholds, it is possible to have confidence in the better series if the items are correctly marked. With the first procedure, the scorer cannot be so certain.

It is a matter of some practical importance to know that the spurt-order in a particular series is immaterial. The same thresholds are obtained when the same spurt-order used in the test is retained in the retest, and when the retest spurt-order is quite at random.

It makes some difference what intensity-level sequence is used. A procedure using 5 db steps is more reliable than one using 10 db steps. Yet a 10 db step procedure can be used as a compromise in the following manner: instead of a sequence 30-20-10-0-omit-10-20-30, 39 subjects were given test-retest on the sequence 30-20-10-omit-0-5-15-25 ("dove-tailed"); these were compared with 33 subjects given the sequence 30-25-20-15-10-

TABLE V. Comparison of reliability of two intensity-level sequences. Entry: Mean deviation of test-retest, ±1 standard error.

	Duplicative scheme		Dovetailed scheme			
	Mean deviation	Standard error	Mean deviation	Standard error	Difference	Standard error (difference)
256	2.25	.38	3.55	.71	1.30	.80
512	1.35	.37	1.90	.46	.55	.61
1024	.20	.11	1.35	.37	1.15	.41
2048	2.40	.37	2.40	.40	0.0	.56
4096	2.75	.50	4.05	.65	.30	.82
8192	3.70	.50	3.10	.72	.60	.88

TABLE VI. Comparison of the manner in which the two tests distribute a population.

	Individual mean acuity loss in db	Group mean acuity loss in db	Difference	Critical ratio	Individual standard deviation	Group standard deviation
256	7.00	6.15	.85	1.7	3.75	5.70
512	1.95	2.30	.35	.7	3.80	5.20
1024	.90	−2.95	3.85	7.0	5.80	5.00
2048	1.15	4.95	3.80	5.0	7.70	7.05
4096	10.50	9.35	1.15	.7	13.85	14.20
8192	6.60	10.15	3.55	2.2	15.50	13.90

5-0-omit-5-10-15-20-25-30 ("duplicative"). Since neither series sampled better-than-normal acuity, results must not be compared directly with earlier tables; reliabilities of Table V are spuriously high because of the restricted range, but can of course be compared one with another.

In general these data show the duplicative scheme to be somewhat more reliable—though this is not strictly true for all frequencies. But if in addition to taking account of the sizes of the mean deviations, one notes also the sizes of the standard errors, it is seen that uniformly more dispersion occurs when the dovetailed scheme is used.

However, since the dovetailed scheme will with 7 items sample every 5 db step from normal to 30 db above normal, whereas the duplicative scheme uses 14 items to produce substantially the same data, we conclude that for some purposes the dovetailed scheme is indicated in the interest of efficiency.

The validity of a group audiogram is ascertained by comparing the thresholds of a subject obtained by a group test with his thresholds obtained by a careful individual test. Table VI presents the mean loss and the standard deviations of acuity for the two tests on the same subjects.

The small differences between mean loss, even though reliable, as well as the correspondence of the standard deviations, are indications that there is no strong tendency for "group" averages to be different from "individual" averages—that is, the tests distribute the population in very much the same manner. If there were, for example, a tendency on the group test to miss the last one or two settings which would be heard on an individual test, such a factor would show in Table VI. But as a matter of fact the group test mean runs somewhat better at three of the six frequencies.

Neither is there a tendency on the part of the group test standard deviations to be larger. There is no strong tendency, making for variability, inherent in the group and not in the individual method.

We wish further to know how widely a particular subject's "group" score may be expected to deviate from his "individual" score. This information is found in Column 1, Table VII.

TABLE VII. Validity of group audiometer. Entry: Mean deviation of first test score from second test score.

	1 *vs.* 2 (validity) (N = 290) mean deviation	1 *vs.* 1 (reliability) (N = 72) mean deviation	Difference	Critical ratio
256	4.45	3.80	.65	1.4
512	3.80	3.35	.65	1.6
1024	3.65	2.05	1.60	4.0
2048	4.25	3.85	.40	.8
4096	5.70	5.10	.60	.7
8192	6.15	5.40	.75	1.1

Legend: 1—Group test.
2—Individual test.

The mean deviations of Column 1 may be regarded as expressing validity. In order quickly to interpret these figures, the mean deviations of group test-retest reliability (Column 4, Table III) have been repeated here, and the usual test for significance applied.

Since the deviations from group-to-individual test are only slightly greater than the deviations from group-to-group test, it seems to make little difference whether a second test is given individually or by groups. Inasmuch as we do not usually look for a higher validity statistic than the reliability of the less reliable test, these validities are probably satisfactory.

Clinical Phenomena in Conductive Media: The Individual Earpiece*

MAYER B. A. SCHIER, D.D.S.
110 East 42nd Street, New York City
(Received June 2, 1945)

A HEARING aid may be well designed and produced, acoustically and mechanically, but its optimum efficiency will depend upon maintenance or supplementation in delivery of instrument characteristics to the drum or middle ear structures. It has long been my contention that commercial earpieces in test and subsequent permanent couplings are sources of error, and very often incorrect; that the coupling units must be individually designed to meet the general pathologic classification of the patient's need—the characteristics of the instrument between mic input and receiver output, and both reconciled, i.e., patient and instrument coupled through a properly fitted earpiece.

In the presentation of a paper as this, it becomes obvious that the approach is in method concomitant to clinical practice, but that a continuity of practice results in conclusions, fundamental in nature. I have assiduously avoided the presumptious utilization of other's work to bolster my observable phenomena. This has been done for three reasons: First, it augurs little good if one possesses integrity. A contribution should be genuinely one's own. The flair for pedancy is in reality nothing more than a veneer to dress up inconsequential worth; Second, I have not always had available the facilities so essential to pure scientific investigation as I shall later exemplify; Third, because I have long believed that the known principles of sound, transmission, and formulae, are based on the normal and inorganic, whereas the problems of the human ear concern the sub-normal and organic,—the processes of which are as yet little understood.

In addition, and for perspective purposes, it is essential to consider generalized pertinancies, to wit: The frequency-response curves of hearing aid or sound testing equipment and that of the ear, are generally in single tone recordings, whereas, in function, sound, and speech are complex in nature; The frequency-response curve of a hearing aid cannot be related, *per se*, to an audiogram of the ear, for one is a proportional rating and the other based on a mean standard. Intelligibility of speech is dependent upon the transmissive fidelity of the instrument; proportional reception and perception of the lows, highs, and their harmonics. The greater the strength of the lows and overtones with increasing losses in the sub-normal ear, the lesser the toleration range of intensity and intelligibility.

The tendency of modern high fidelity vacuum-tube instruments is towards a basic circuit encompassing over-all amplification which permits a maintenance of the highs with provisions for attenuating the lows in successive frequency bands, presumably to meet the requirements of a particular sub-normal ear.

Hence, any evaluation of the precise effects of an earpiece must include considerations of the pure tones and overtones, and of the intensities upon the tones as complexities and not alone as fundamentals.

For the purposes of avoiding confusion and to review as much as possible of the subject within time limitations, I have discarded chronological sequence in favor of oriented divisional considerations. In sum, the productive and alterative effects which may favorably or adversely induce minimal or maximal correlations between the instrument and deafened ear by changes in the intensity, pitch, and timbre of sound, will be discussed.

To digress for a moment, my earliest experiences in utilizing a transducer of sorts, for the collection and concentration of sound, and its deposition at the tympanum occurred some twenty-five years ago. To avoid cupping of ear for long range reception and to facilitate taking notes, a mechanical crutch was attempted. From a modelling compound impression of the ear, a vulcanite reproduction was made. The mass of the body was hollowed out and a wide open canopy vulcanized on to it (Fig. 1). In some instances it worked well, in others, not. Another earpiece was

* Delivered before the Acoustical Society of America, May Meeting, Hotel Pennsylvania, New York City, May 12th, 1945.

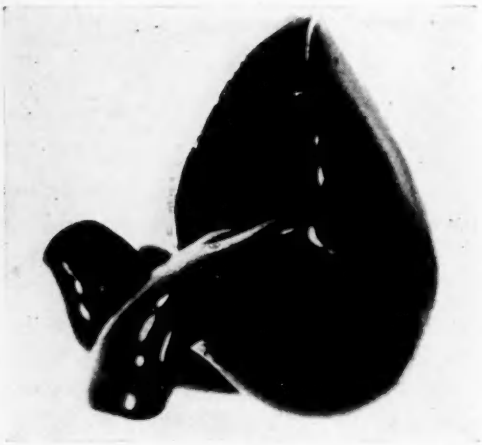


FIG. 1.

made with a rather flat and closed canopy (Fig. 2). This proved superior under conditions where the first was less effective. As I can recall, in the smaller lecture room, the high flanged open canopy served well, but in the amphitheatre, with its square, bare openness and high ceiling, and seated well above and away from the lecturer, the flattened canopy earpiece proved advantageous. Significant is the fact that the aural condition was diagnosed as oto-sclerosis.

Though variant attempts had been made with the old flat-type receivers through the adaptation of meatal conduits, nothing was so conducive to the study of this problem as the introduction of the miniature receiver. In the twenty and early thirties, air-conduction hearing aids were fitted by means of commercially molded right- and left-stock earpieces, running in five or six numbers. The individual earpiece made from an impression of the ear was not an unknown quantity. It was regarded as necessary *only* when the commercial earpiece proved inadequate for retention within a particular ear. Up to approximately twelve years ago, the earpiece was solely an expedient for retaining the miniature receiver in the ear and *not* because of any acoustic or conductive import. It was to no avail, more than fifteen years ago, that I tried to induce successive major manufacturers and laboratories to consider the retentive, acoustic and conductive importance of a properly fitted earpiece and its bearing upon instrument and the deafened ear. The introduction of the bone oscillator in 1932 afforded an opportunity to demonstrate the fundamental worth of the individual earpiece. I believed and stated then, that bone conduction was not the medium of choice, that an air-conduction instrument with an earpiece individually fitted would prove superior to bone conduction. This challenge was finally accepted by the president of a leading hearing aid company. The results were very positive, and so was initiated the mandatory practice of having an individual earpiece as standard equipment for every air conduction aid. This was subsequently followed by the proponent of bone conduction, until to-day, it is an acceptable and generalized practice.

First. The simplest consideration is that of actual coupling of a receiver to earpiece. Insecurity or instability results in losses in intensity, intelligibility, and timbre. Where considerable battery power is used and selectiveness of instrument is for higher frequency amplification, backlog and squeals are set up. It appears that the free-play of receiver nib initiates disturbances in the sound line.

Experimentally, four tests are made. A thin washer is interposed between receiver and earpiece surface to take up slack. This is removed and replaced by another which has been cut in

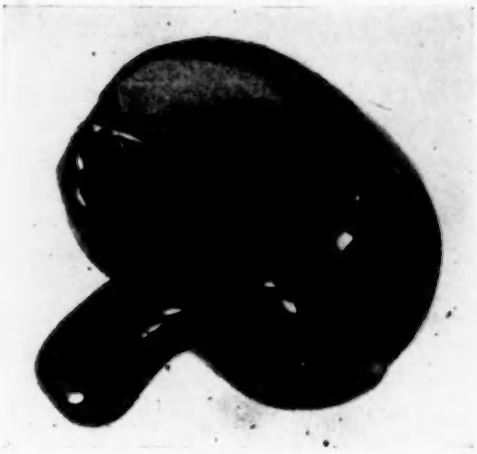


FIG. 2.

pieces and spaced over the earpiece collar or bushing, and then coupled; then again, the washer is removed and the receiver waxed directly to earpiece; and finally, the bushing is adjusted to insure unaided adequate coupling. In the main,

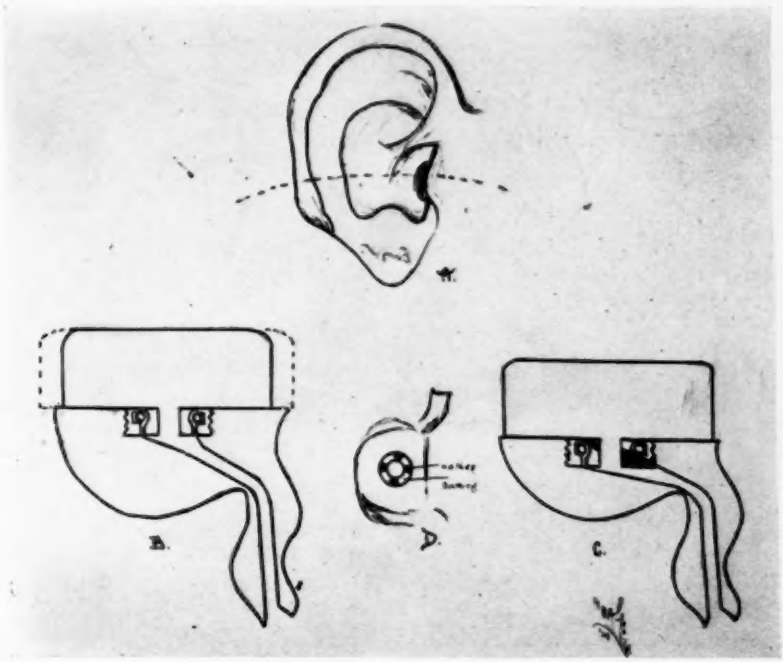


FIG. 3. *A*: Cross-section area of ear schematically presented in subsequent figures. *B* and *C*: Different-sized receivers and relationships to earpiece as well as effects through design on sound channels, etc. *D*: Spaced-cut washer over earpiece bushing.

though there are admitted exceptions, sound conduction appears as consistent as the patient can determine through conversational checks (Fig. 3).

Second. A, the body proper. In the early days of vulcanite, in spite of engineering beliefs and lack of laboratory interest, clinical experiences were definite in that the degree and thoroughness of vulcanization of hard or soft rubbers demonstrably influenced sound conduction. It varied in extremes from attenuation to marked sharpness. In recognition of this phenomenon, and the difficulty of uniform production control for consistent homogeneous body in variable masses, research led eventually into the field of plastics, and my development and introduction into the hearing aid industry of the now universally used acrylic type earpiece. The acrylics are of such nature as to result consistently in uniform molecular densities in routine production and of passive nature. The low Brinnell hardness number may explain its satisfaction as a media material.

Second. B, cavities within the body of an earpiece. As a routine procedure for generalized practice, it is rather difficult. Sound and speech are favorably increased in cavo-formed earpieces of various dimensions. Not unlikely a combination of reflection and aberration of the original sound and the tolerances to distortion in predominant middle ear or obstructive deafness, may be the clue to this favorable reaction (Fig. 4*A*).

Second. C, air content within body. Intervention of the pressure or confinement of air within the sound-conducting cavity below nib of receiver and channel of earpiece has been utilized as a factor for variable sound effects. Much will depend upon an accurate knowledge of the instrument, particularly its selective amplification, amount of power used, length of actual tip, and diameter of sound bore. A unilateral opening extending from exposed or partially exposed top surfaces of an earpiece down to the sound channel can prove of value. We have utilized two methods of approach for similar intent. One has a variable valve control, and another by the use of successively increased reamed openings. Through conversational tests an effort is made to determine which sized opening proves advantageous (Fig. 4*C*).

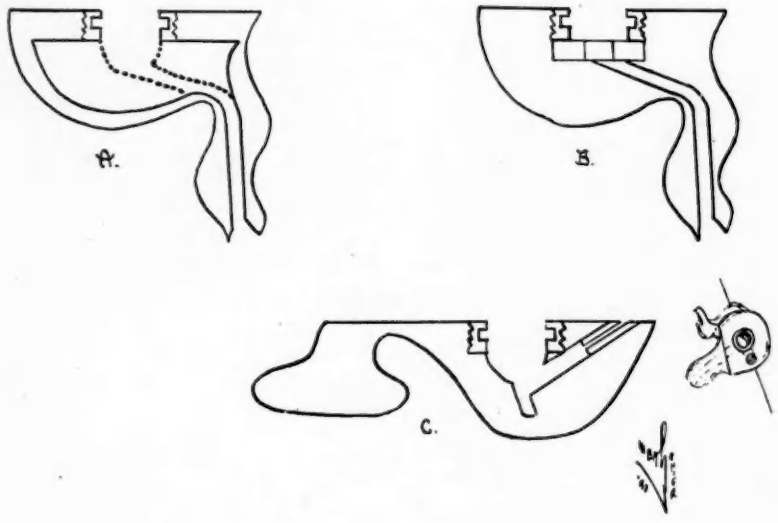


FIG. 4. *A*: Cavities within body of earpiece—from large outline to dimensions approximating dotted form. *B*: Filtration or by-passing with acoustic chamber device between bushing and continuing sound channel. *C*: One form of varying air content with lateral opening from surface to area below bushing and continuing sound channel.

I would have liked to enlarge on this and the subsequent subdivision. Perhaps at some early day, there may be the opportunity of submitting a particular method and preparation for analysis and so possibly explain the favors to perceptive deficiencies, *q.e.d.*, the higher frequencies. These studies would include clinical, acoustic, and frequency complexities.

Second. D, filtration and by-passing within the body. Intervention by filtration and interference within the acoustic path has demonstrated advantages. A little device composed of miniature acoustic chambers is inserted within the line between receiver nib and sound channel. The frequency-response curve of an instrument can be so effected as to modify the relativity of the low, medium, and high frequencies as emitted from receiver. Laboratory co-operation confirming clinical experiences was obtainable. These devices were inserted within little tubes in simulation of earpiece sound channels. To quote briefly from a letter[1] of record, "regarding the tests which were made for you in April 1939 . . . an original chart showing four frequency-response curves designated Std. .6, 1, 11, 111, . . . all these curves were taken with a representative .6 micro-midget receiver feeding a 2 cc artificial ear. That the receiver was operated from a constant voltage variable frequency source and that the curves designated by 1, 11, 111, had three variations of

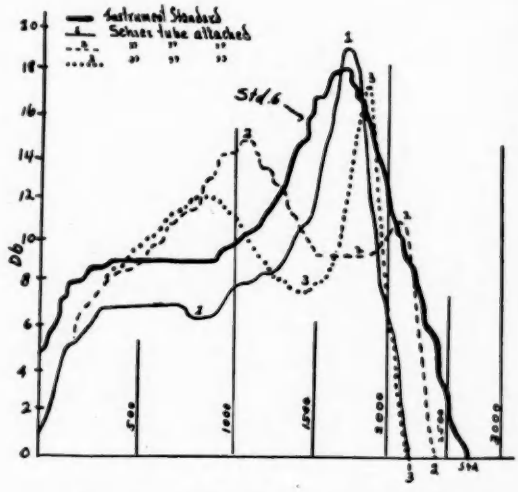


FIG. 5. Four frequency-response curves of same instrument; Std. 6, micro-midget receiver; 1, 2, 3, Schier attachment or chambered device interposed.

[1] Letter from Mr. Dean Babbitt, President of Sonotone Corporation, through whose courtesy the company laboratory facilities were made available; and to Mr. H. A. Pearson and Dr. H. W. Koren, engineer and physicist, who kindly performed the tests and certified the original from which the graphs shown in Fig. 5 were made.

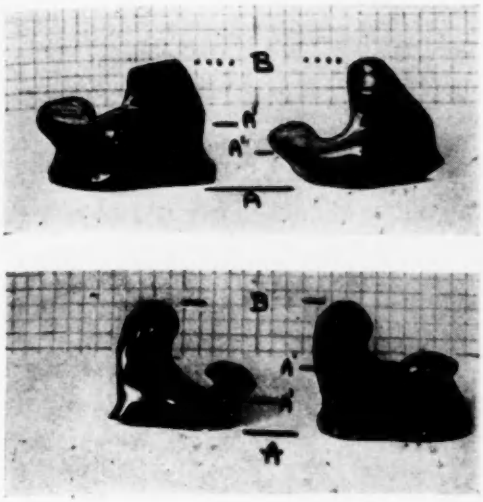


FIG. 6. Comparative presentations of the same ear. The highly polished ones are the author's. The body bulk has been reduced by a half and more. *A* to A^1, the tip lengths, are almost twice and follow the turns of the canal proper.

your attachment interposed between the micro-midget receiver and the artificial ear" (Fig. 5).

Of significant interest is the fact that the acoustic interceptors placed in the feed line, and giving the different results shown by response curves 11 and 111, are one and the same. The difference in effect has been merely due to reversing the imput and output ends of the device in relation to receiver nib and continuing sound conduit (Fig. 4*B*).

There appears to be a paradox in the last two subdivisions, at least from pure physical reasoning or experiment. I am not convinced as to the relative independent importance and explanations given for confined air, resonance, and pressure, though this last has been strongly ventured as the incidence essential to ear deficiencies requiring higher frequency carriage.

Third. A, length of tip and meatal form. The external auditory meatus or ear canal does not run to the pattern of anatomical texts, nor is it completely appreciable to otological examination. Viewed in the horizontal, it runs the gamut of variable length, in round to ovalar forms, from near atresia to the cavernous, from simple undulating double curve to extremes in sinuosity, from progressive narrowing diameters to irregular cavity forms within its length. To the deafened ear these forms project definite problems in sound passage. They will induce variables of impedance, resonance, absorbtion, and so forth.

It is my belief that to the subnormal ear, ever so much more than the norm, these factors inherent and effectuated by meatal form can be,

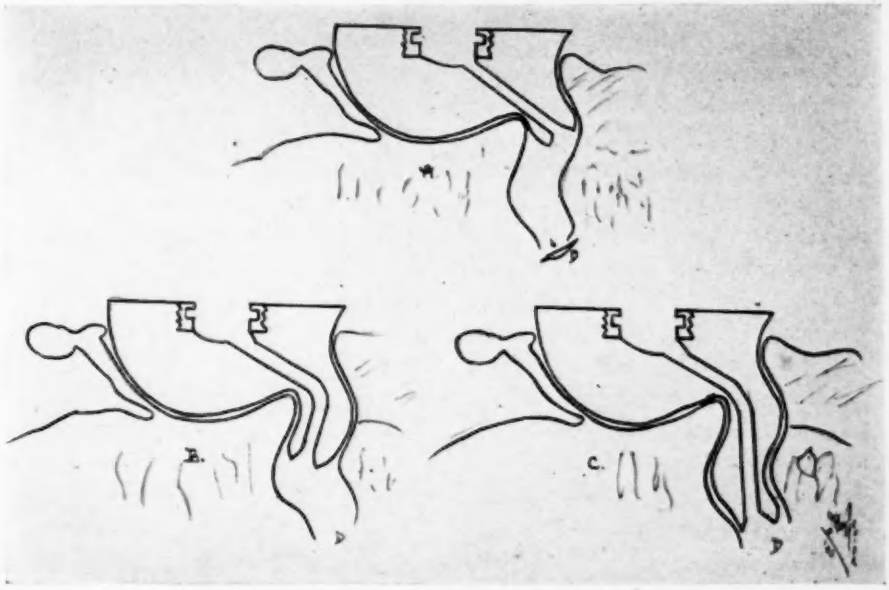


FIG. 7. *A*: Cross section of earpieces within auricle and tip relationships to canal—short tip just below lip of meatus. *B*: Below lip or first bend but short of second. *C*: Long tip beyond both bends and in line with tympanum; successive presentations of orifice relationship to meatus walls and obvious effects on emitted sound.

and should be, mostly eliminated. Hence, I've recommended a direct acoustic conduit between receiver to middle ear structures. In proportion to the length limitations of the actual tip, and its relation to meatus, will the instrument be negated and the ear suffer not alone as to transducer characteristics but through sound blocks, muffling, resonance, back-log, or squeals, etc. Typical of misrepresentation of approaches I have advocated is that concerning the use of a tip as long as the patient can reasonably tolerate. This is not an old practice which has been recently popularized by me. It is an old practice which long antedates my paper in the Laryngoscope of 1941 and likewise that of a professional lecture-clinic and paper published in 1933 from which Fig. 6 has been taken.

Figure 7 is a schematic presentation of the ear in cross section, with earpieces of different length tips. It is to portray the obvious possible effects of tip openings in their meatal relationships, and the actual sought for repository, the tympanum or middle ear.

Third. B, diameters of sound channel. Considerable clinical studies and observations have been made with different sound conduits from the finest to 30 gauge. Maximum diameters are desirable in keeping with modern high frequency concepts. It has become a practice, for purposes of establishing a mean or standard, to utilize a 42-gauge drill as the optimum in routine productions, the earpiece and canal allowing.

Third. C, tip end to tympanum. In the Meeting at Rochester in 1942 I raised this question of phenomenon, but no answer was made then, or afterwards, though the question has been repeatedly discussed. It is a fact, that you can make an earpiece as long of tip as the patient can tolerate. You may check carefully for maximum relationship between tip orifice and tympanum. Yet, pressure upon receiver markedly increases both intensity and intelligibility. It must be borne in mind that the actually added ingress of tip can hardly equal $\frac{1}{16}$ of an inch. Building up the tip not alone becomes physically intolerable, but does not give the same results.

Whether or not it is due to a compression of the small air cavity between tip end and tympanum, with pressure as the incident,—or whether to the subnormal ear it is a small but powerfully effective manifestation of the Doppler principle, remains for investigation.

THE JOURNAL OF THE ACOUSTICAL SOCIETY OF AMERICA VOLUME 17, NUMBER 1 JULY, 1945

Musicology, the Stepchild of the Sciences

ABE PEPINSKY
Haverford College, Haverford, Pennsylvania
(Received May 29, 1945)

WHEN our editor asked me to review the Proceedings of the 1939 International Congress of Musicology, he timidly suggested that our journal ought to include such a review inasmuch as musicology purports to be the scientific study of music. Webster's Dictionary seems to support such a premise, for under musicology is found: "Music as a branch of knowledge or field of investigation; especially, historical study of musical documents, investigation of sources, gathering and organization of neglected data." This, however, is a rather restricted sense of the term, for in actual practice musicology reaches into many fields, and it has been facetiously remarked that it includes practically everything that lies outside of the personality of the performer and the purely technical aspects of his performance. The English scholars prefer to translate the original German term *Musikwissenschaft* as musical science; but they limit the science of music primarily to physical acoustics, and only occasionally include physiology and psychology as related to music. There has been quite a change of attitude on the part of the British scientists of late, however. This is especially true of Alexander Wood in his latest book on the Physics of Music, wherein he even includes a consideration of esthetics and musical expression.

The history of musicological endeavor shows that the science of music has ever been a contemporary of the art of music. Pythagoras was evidently imbued with the spirit of musicological research in working out a mathematico-acoustical basis for the measurement and comparison of musical intervals. From the 6th century B.C. until the 2nd century of the Christian era a number of Greek philosophers and theorists developed increasingly comprehensive systems of musical knowledge. Quintilianus suggested a system which evidences a surprisingly sophisticated apperception of the musicological approach. He partitions his treatise into a theoretical and a practical part. The theoretical part is further divided into a physical or scientific section which treats of arithmetic and physics, and a technical section dealing with harmony, rhythm, and meter. The practical part is also divided into two sections, one of which has to do with melodic, rhythmic, and poetic composition; and the other, on execution, dealing with instrumental, vocal, and dramatic performance. This catalog and the accompanying discussion reveals a remarkable insight into some of the problems involved in the organization of musical knowledge. Boethius, early in the 6th century, includes music in the quadrivium, the four branches of mathematical learning—arithmetic, geometry, astronomy, and music. Cassiodorus, a few years later, treats of music under the divisions of a harmonic, rhythmic, and metric science. These writers obviously reflect the philosophies of the ages they represent, and their discussions of problems of practical musical instruction are quite unsystematically intermixed with theoretical speculations. One aspect is conspicuous for its absence throughout the Middle Ages, namely the historical viewpoint. However, the 17th and 18th centuries offer an ever increasing evidence of this important branch of musicology. Finally, in the late 19th century, Guido Adler incorporated both the systematic and the historical viewpoints in a comprehensive statement of the scope, method, and aim of musicology. This attitude of approach was likewise followed by Hugo Riemann and Volbach. The "higher" studies of the Middle Ages included music, at first in the monastic and cathedral schools, and later in the universities. This was especially true of the English universities after the 16th century. The chairs of music were discontinued for a time on the Continent, but were re-established in the late 19th century. In our country, a number of the more important universities are offering formal graduate courses in the various fields of musical research.

Musicology, evidently, has not been a totally neglected stepchild of the sciences, although

somewhat embarrassing to them at times because of the discrepancies between the theory and actual practice of music. The subdivisions of a systematic musicology, as differentiated from an historical approach, are based on both logic and convenience. Acoustics attempts a descriptive analysis of music in its physical nature. Psychophysiology treats of problems related to music as a phase of man's behavior. Esthetics concerns itself with questions of value in music. Anthropology deals with music as an element in the sociocultural make-up of the various peoples of the world. Pedagogy treats of problems of music education. All contribute, each in its own characteristic way, to the more intrinsically musical science of music theory. Such theory is the direct study of the artwork itself and attempts the analysis, if not the explanation, of the musical composition in technical terms.

When the science of music begins to scrutinize the music itself, the musical stepchild becomes especially embarrassing to the musician. The musician is apt to frown upon the musicologist because, although the musicologist is supposed to know all about music, he seems to derive no sensory pleasure from the performance of music. Acoustics, psychophysiology, esthetics, and anthropology are not ordinarily a part of either the general musician's training or the formal college curriculum in music. The university's music department is often but little more than a glorified conservatory with a faculty that earn their salaries largely by selling piano, voice, or violin lessons. It is a paying proposition for the parent institution because a definite charge is made for so-called "over-head" against the special fees paid by the students for their lessons. It must be confessed that the music student's academic background is usually not a very scientific one. He, therefore, does not feel encouraged to go on to graduate fields majoring in musicology. It is much easier to entice him with the bait of an advanced degree with but little more required of him than the expansion of a movement of a pianoforte sonata for orchestra or band. One other possible goal for such a student is to be found in the field of music education. But the average public school music teacher has far too anemic a musical background to do serious research in musicology. Who then is interested in musicology? Is it the disappointed musician who has been unable to acquire the necessary technical facility for a successful concert career and does not wish to be a music teacher?

There is one field of musicological endeavor that seems to tempt the musician. This field might be called historical musicology and consists of the philosophy of music history, the sources of musical history, and the problems and methods of historical research in music. The historical orientation offers an interpretation of the experience of the individual in relation to that of the race, and in relation to the realities with which he has to deal. In musicology, this generalization means that the historical mode of interpretation, in terms of temporal sequence, supplements the systematic in terms of quality and structure. Such an approach can be very intriguing to the musician lacking the requisite ego of a performer or composer but who may develop a critical attitude. The systematic and historical approaches are not at all incompatible, but constitute the two axes in the frame of reference in relation to which musical intelligence is oriented. The two approaches are complementary; the one cannot be maintained without reference to the other. And so our candidate for graduate work in music may be introduced to fields in which he might have otherwise had no interest. The individual items of musical experience, whether scale, folk-song, or symphony, can be illuminated for human intelligence only by being placed in the proper historical as well as systematic frame of reference. To determine what constitutes the proper frame of reference is naturally a fundamental concern of musicology.

If I were to enumerate the subject matter generally considered by the musicologist under the caption of systematic musicology, it would read very much like the cumulative index of papers in the section on musical instruments and music of the *Journal of the Acoustical Society*. In fact, that is just where the reports of the more significant research in the field of systematic musicology actually appeared, since 1929, at least. It must not be thought that any one musicologist ordinarily busies himself with all these many attributes of scientific research in music. But his keen appreciation and interest

may well serve to stimulate further the specialist in science. For instance, the field of acoustics as a branch of musicology is considerably narrower than the general field of acoustics. The application of the theories of sound production and transmission brings about a division of the field according to the interests served. Although the results of research are of great importance to the musician and he shows a readiness to cooperate, he is perfectly willing to leave to the physicist and the architect the study of the acoustical properties of rooms and auditoriums. The design and mechanical development of musical instruments are often stimulated by the technical demands of a composer, but the actual research is again done by specialists and artisans who learn their trade by the apprentice method. As a part of music theory, there can be no question of the great importance of the acoustical implications in the consideration of systems of chord classification, in problems of intonation, temperament, principles of orchestration, and even in connection with the actual performance of music. Acoustical principles are also involved in the playing of instruments and singing. The venting on woodwind instruments, the effects of embouchure or "lipping" of a brass instrument, the use of the vibrato, the perennial discussion of the differentiation in the manner of striking the keys of the claviature type of instrument, diction, and many other similar problems are all, of course, largely acoustical in nature. Problems of transmission related to the radio, sound films, and in the phonographic reproduction of music fall mainly to the lot of the electrical engineer, but can be greatly aided by the experience and interest of the musicologist. It would be desirable, therefore, that a well-equipped acoustical laboratory be available in every well-rounded department of musicology for special research projects.

It is obvious that the consideration of music must deal with its effects upon the human organism as well as upon the objectively measurable physical findings. Anatomy, physiology, and psychology reveal many principles of significant interest to the musicologist. The structure of the human body has played an important determinative role in the development of music, in the shape and the dimensions of musical instruments, as well as in the acquisition of adequate technical performance on them. The study of the central nervous system connecting the sensory organs and the motor apparatus offers the possibility of a physio-psychological consideration of musical performance. Knowledge of the structure and functioning of the end-organ of hearing, its non-linearity and asymmetry, helps to explain the difference between what the composer writes and what the listener actually hears in the performance of music. The notion of the pitch of a musical note is of fundamental importance, and it is recognizably affected by changes of intensity, timbre, and type of vibrato. Pitch is also differently conceived when melodically and harmonically considered. The present generation of musicians has also greatly stressed the notion of the so-called "expressive intonation," whereby the pitch of a tendency tone is exaggerated in its approximation of the rest-tones of a melodic sequence. Due to the psychophysiological limits for pitch discrimination, the wishful thinking of some of our ultra-modern composers may be greatly curtailed when subdividing the octave into ever smaller steps—even one-sixteenth tones. The subject of absolute pitch has stimulated a very considerable literature, as has also the synesthetic relations of tone, color, and odor. Many musicians have trained themselves for an adequate tonal memory, something like a sense of absolute pitch, but the fact remains that the same ability is shown by otherwise unmusical people. It is a gift of sometimes doubtful value. The very extensive literature on the vibrato as a musical ornament has somewhat squelched those who would have none of it because it might cover a multitude of sins of intonation, and because in the voice it is often confused with an unwanted tremolo. For a time it was assumed that the principal pitch of a tone with a frequency vibrato was the geometric mean tone. But newer studies indicate that for extents less than a quarter of tone, the principal pitch is slightly above the geometric mean, while for a vibrato greater than 0.6 of a tone, it is lower. Closely related to the vibrato is the intertone which is heard when two beating primary tones differ in frequency by only a few vibrations. Other subjective-tone phenomena have found practical applications as well as

stimulus for renewed research. One of the most meaningful contributions to a better understanding of instrumentation problems is the study of masking effects. Although the more pertinent results are those based upon the use of pure tones, some consideration has been given to modification due to complex tones. It is to be hoped that such studies may be further pursued. The effects of intensity and phase, so important in binaural hearing, have been used lately in a novel type of entertainment under the caption of stereophonic sound. Something more pertinent can well be expected in this field. Acuity for temporal values has been tested along with various other measures of musical capacity, and the minimum perceptible difference in the length of tones varies from 0.01 second for a good musical ear to 0.2 second for an inferior ear; but the effects of musical transients at threshold temporal values have yet to be more thoroughly explored and evaluated. The musical interval, conceived both harmonically and melodically, has been a tantalizing toy of the Gestalt psychologist as a fundamental unit of perception. The concept of a melodic interval, as having a kind of kinetic energy has been quite fruitful in certain aspects of a theory of musical esthetics. The arbitrary nature of many musical scales has been treated from a number of points of view. It has long been the plaything of the mathematician. Leibnitz even defined music as the subconscious exercise of the soul in simple arithmetic. The field of comparative musicology has long been interested in the music of primitive man and that of the Orient and other non-occidental peoples. The organization of psychologically discriminable intervals into familiar patterns used in music has offered large reward to the research worker in esthetics.

One might go on interminably with the listing of scientific exploits in the musicological field, but we must cry halt in so brief a survey. The scientist has frequently pampered the musicological stepchild. Indeed it would seem that the scientist has himself often taken over the functions of a musicologist; but it is to be hoped that the musicologist of the future may be sufficiently trained as a scientist, to better work out his own salvation. Our stepchild, grown to full maturity, should then be able to take his place in the world of science as a respected citizen.

Letter to the Editor *

CHARLES WILLIAMSON
Schenley Park, Pittsburgh, Pennsylvania
April 10, 1945

IN a recent paper[1] I made the following remark: "Many competent observers incline to the belief that the foremost string and vocal ensembles tend to perform in just intonation; but the complicated and difficult task of testing their music quantitatively has apparently not been attempted." Young[2] has listed this investigation as a problem for postwar musical acoustics.

An investigator who has access to the facilities of a modern transcription studio might be able to measure the intonation of a string quartet by attaching a contact microphone to each of the instruments and making four simultaneous recordings. Each recording could then be tested at leisure by methods applicable to single instruments.[3] The usual stroboscopic test would insure turntable synchronism during recording and reproduction. Similar tests of brass, woodwind, or vocal quartets might be made with the aid of four extremely directional studio microphones.

* The Editorial Board assumes no responsibility for the opinions expressed in letters.
[1] Williamson, Am. J. Phys. **10**, 171 (1942).
[2] R. W. Young, Acous. Soc. Am. **16**, 103 (1944).
[3] Reference 1, footnote 16.

Acoustical Society News

May, 1945 Meeting in New York

The New York meeting in the Hotel Pennsylvania on May 11 and 12 was well attended, considering the restriction on the travel of out-of-town members which was administered for ODT by Secretary Waterfall. The meeting included symposia on fundamental acoustic measurements and quality requirements in sound systems.

Dr. Leo L. Beranek added the Chairmanship of the Program Committee to his many other duties and the Society is indebted to him and the members of his committee for preparing an excellent program. The members of the committee were J. E. Volkmann, E. C. Wente, Franz N. D. Kurie, E. J. Abbott, and E. Dietze.

Move

The Sound Apparatus Company, formerly of 150 West 46th Street, New York City, has now moved to 233 Broadway, Woolworth Building, New York 7, New York.

New Members

Athol M. Anstey
6 Bradley's Head Road
Mosman
New South Wales, Australia
Head of Sonotone Hearing Aid Dept.
Angus and Coote Pty. Ltd.
500 George Street
Sydney, Australia

Mrs. M. Hildred Blewett
General Engineering and Consulting Lab.
General Electric Company
Schenectady 5, New York
Physicist

Bruce Plympton Bogert
1150 Fifth Avenue
New York, New York
Physicist
Radiation Laboratory, Massachusetts Institute of Technology
Cambridge 39, Massachusetts

Marvin Camras
1313 South Keeler Avenue
Chicago 23, Illinois
Research Physicist
Armour Research Foundation
35 W. 33rd Street
Chicago 16, Illinois

Henry C. Dalrymple
46 Colony Drive East
West Orange, New Jersey
Engineer
Guided Radio Corporation
161 Sixth Avenue
New York 13, New York

David J. Evans
1505 Rosecrans
San Diego 6, California
Physicist
University of California
Division of War Research
San Diego 52, California

James L. Fouch
617 Exton Avenue
Inglewood, California
Universal Microphone Company
Business Executive-Engineer
424 Warren Lane
Inglewood, California

Wilbur T. Harris
Box 10
Southington, Connecticut
Physicist
United States Time Corporation
Waterbury, Connecticut

Herbert Arthur Harvey
319 Gowan Avenue
Toronto 6, Canada
Engineer (Radio Tube)
Radio Valve Company
189 Dufferin Street
Toronto, Canada

Denison B. Hull
77 West Washington Street
Chicago 2, Illinois
Architect
President, OTARION, Inc.
Mfgs. of Hearing Aids
448 N. Wells Street
Chicago 10, Illinois

Robert S. John
809 N. County Street
Waukegan, Illinois
Research Engineer
Pfanstiehl Chemical Co.
Waukegan, Illinois

Orin Benjamin Johnston
6730 Harriet Avenue South
Minneapolis 9, Minnesota
Engineer
Minneapolis-Honeywell Regulator Company
2753 4th Avenue South
Minneapolis, Minnesota

M. O. Kappler
4784 Coronado Avenue
San Diego 7, California
Electronics Engineer
Univ. of Calif. Div. War Research
San Diego, California

Alec Herbert King
Hast Hill
Hayes, Bromley
Kent, England
Physicist
Research Laboratory
British Thomson-Houston Company, Ltd.
Rugby, England

George Kis
827—4th Street
Santa Monica, California
Project Engineer
Frank Rieber, Inc.
11916 W. Pico Boulevard
Los Angeles 34, California

Homer Charles Knauss
30 Lancaster Street
Cambridge 40, Massachusetts
Research Physicist
Submarine Signal Company
Boston, Massachusetts

Armand F. Knoblaugh
6408 Cary Avenue
Cincinnati 24, Ohio
Physicist and Engineer
The Baldwin Company
1801 Gilbert Avenue
Cincinnati 2, Ohio

William G. Langton
Randall Laboratory of Physics
University of Michigan
Ann Arbor, Michigan
Research in Supersonics

Sol Levine
805 Street and Marks Ave.
Brooklyn 14, New York
Project Engineer and Physicist
Bendix Aviation Corporation
Marine Division
Brooklyn, New York

Robert Livingstone
2066 Hyde Park Blvd.
Los Angeles 44, California
Manufacturing Sluccos and Acoustic Plasters
Velvatone Stucco Products Company

Charles S. Minor, Jr.
813 Martin Bldg.
Birmingham, Alabama
Architectural Acoustics
813 Martin Bldg.
Acoustic Engineering Company
Birmingham, Alabama

Howard R. Molton
1459 West 71st Street
Chicago 36, Illinois
Physicist
Russell Electric Company
340 W. Huron Street
Chicago, Illinois

Howard K. Morgan
6447 Sagamore Street
Kansas City 5, Missouri
Radio Engineer
T.W.A. Inc.
10 Richards Road
Kansas City 6, Missouri

William John Muldoon
10 Washington Street
Hempstead, New York
Engineer
Guided Radio, Inc.
161 Sixth Avenue
New York 13, New York

Wesley L. Nyborg
172 Hartswick
State College, Pennsylvania
Research Assistant in Physics
Acoustic Department
Physics Department
State College, Pennsylvania

Roscoe W. Palmer
2222 North Laurel Avenue
Phoenix, Arizona
Hearing Aid Consultant

Hubert Beaufort Peugnet
AAF School of Aviation Medicine
Randolph Field, Texas
Major, Medical Corps. AUS
(Research, Department of Otolaryngology)

Fritz Pordes
5216 Drexel Avenue
Chicago, Illinois
Chief Research Engineer
Russell Electric Company
340 W. Huron Street
Chicago, Illinois

Lt. Harry L. Robin
9402 T S U Sig C
Army Experimental Station
Pine Camp, New York

Edward Shulman
Hearing Clinic A-5
Borden General Hospital
Chickasha, Oklahoma
Hearing Rehabilitation and Speech Pathology

Frederick Burge Simmons
4 Melbourne Road
Milton 86, Massachusetts
Sales Manager, Hearing Aid Tube Division
Raytheon Manufacturing Company
55 Chapel Street
Newton, Massachusetts

Hulburt Chapman Tittle
92 Merrymount Circle
Buffalo 21, New York
Chief Radio Engineer
Colonial Radio Corporation
254 Rano Street
Buffalo 7, New York

Leslie Townsend
American Franklin Olean Tile Company
Lansdale, Pennsylvania
Ceramic Engineer

LeRoy D. Weld
1838 Blake Blvd. SE
Cedar Rapids, Iowa
Prof. Emeritus of Physics
Coe College
Cedar Rapids, Iowa

Winston Wells
307 East 44th Street
New York, New York
Acoustical Engineer and Designer
Supersonics Inc.
231 East 47th Street
New York, New York

Paul Alfred Williams
716 Masonic Avenue
San Francisco 17, California
Audio Design Engineer
1375 Mission Street
San Francisco, California

Charles Williamson
Physics Department
Carnegie Institute of Technology
Pittsburgh 21, Pennsylvania
Teacher, ware research worker, consultant

Lyle Frederick Yerges
7751 Hood Avenue
Chicago 31, Illinois
Assistant Mdse. Mgr. Sound Control
U. S. Gypsum Company
300 West Adams Street
Chicago 6, Illinois

King Ross
54–32 64th Street
Maspeth, Long Island, New York
Musician; Instrument Design, Acoustical Research

Fellows	179
Members	896
Total Membership	1075

Current Publications on Acoustics

F. A. FIRESTONE
3059 Randall Laboratory, University of Michigan, Ann Arbor, Michigan

Book Review

International Congress of Musicology. Papers Read at the International Congress of Musicology held at New York, September 11th to 16th, 1939. Pp. 301+viii. Music Educators' National Conference for the American Musicological Society, New York, 1944.

This congress was the first ever held in the United States. Three previous international meetings, attended by American delegates, were held at Liége, Belgium (1930), Cambridge, England (1933), and Barcelona, Spain, in 1936. These festivals marked important milestones in the history of musical scholarship. The congress of 1939 was held in this country because the musical program which was scheduled to take place at the New York World's Fair induced the members of the society to plan a scientific meeting in connection with it. The musical festival at the fair was abandoned but the Musicological Congress went on just the same, despite cancellations, financial handicaps, and World War II. The Congress received the indorsement of our Government, and invitations to the foreign delegates were issued through the State Department at Washington. The Carnegie Corporation and the Pan American Union were also helpful in bringing a number of the delegates here, and in organizing the Congress. The American Council of Learned Societies and the Music Educators' National Conference gave indispensable assistance in the publication of the papers read at the Congress.

The question may well be asked why the readers of the Acoustical Journal should be at all interested in a review of the papers read at such a Congress of Musicology, for the term musicology arouses but little favorable attention from either the scientist or the musician. The name has too often literally covered a multitude of sins. The present group of papers are, however, of a distinctly higher level. They also offer some rather pertinent definitions of the term musicology.

The study of musicology goes into many fields and practically includes everything that lies outside the personality of the performer and the purely technical aspects of his performance. Carleton Sprague Smith, in his welcoming remarks at the Congress, answers the question "What is Musicology?" by the brief statement that "it is primarily the scientific study of music." With this definition in mind, your reviewer thumbed through the twenty-five papers read at the Congress and was rewarded by finding a group presented in a session on Music and Science, presided over by our own Dayton C. Miller (who died in 1941). These papers, then, should suffice for admission credentials to a review in our Journal.

Mr. Smith, the President of the American Musicological Society, takes a keen delight in looking askance at the type of musicology covered by the popular concept of an appreciation of music. He deplores the fact that musical professors (!) with scholarly standards and a scientific approach to the subject have been the exception rather than the rule. Music is too often regarded as an indeterminate thing that should be appreciated but not learned. There are classes everywhere on "The Appreciation of Music." "Can one imagine," he asks, "a professor teaching 'The Appreciation of Chemistry'?" Mr. Smith stresses the experimental method (cause and effect) as a sound way of learning and of teaching music, and attempts to explain the evolution of music. He advocates the carrying out of research, just as does a scientist in his laboratory. He calls attention to the fact that Thomas Edison made one of history's most extraordinary musicological discoveries, the phonograph, which is an extremely important supplementary tool of the musicologist today. Mr. Smith likens it to an anaesthetic, for it enables one to put a symphony on an operating table for a certain length of time, without having it disappear into space.

The encouraging feature of Professor D. C. Miller's interest in the Musicological Congress was further stimulated by, at least, one scientifically minded musician, namely Otto Ortmann, in the symposium representing Music and Science. Professor Ortmann read a paper on the "Psychology of Tone Quality," which dealt primarily with the adjective terminology pertaining to the timbre of musical sounds. He further emphasized a point (already stressed in the Acoustical Journal's contributions) referring to the convenient generalizations of tone quality in the various instruments and the voice; pointing out the fact that in most instances such quality varies widely even for any one instrument or voice. He calls attention also to the lack of parallelism between the physical frequency-intensity relation and the psychological pitch-loudness relation. Mr. Ortmann attributes this to the changes in pitch alone resulting in changes in quality for the ear itself, to say nothing of the change in end-organ reaction encountered as one approaches the limits of any sensorial series. He calls attention to an additional variation introduced by the particular sensitivity of the ear to a pitch region corresponding to the meatus resonance of the ear itself.

No musical discussion of a quasi-scientific nature would be quite complete without some reference to the "vibrato." Mr. Ortmann treats this interesting phenomenon of periodic variations of pitch as variations of tone quality. He speaks of a fusion into a single, new quality. He might have labelled this effect "sonance," giving credit to Metfessel and Seashore; but no sources or bibliography were given. It was as if all these findings were attributable to the Research Department of the Peabody Conservatory of

Baltimore. Mr. Ortmann was the director of this institution at the time; but this paper was unfortunately in the nature of a "swan-song," for Mr. Ortmann left the institution soon thereafter. This was a sad blow to musicology in this country, for we haven't heard from Ortmann since that time.

The controversial nature of the theories of harmony was ably presented by Professor Glen Haydon, of the University of North Carolina, in a paper entitled "Alfred Day and the Theory of Harmony." Dr. Day, an English physician of the first half of the 19th century, offered some very significant comments on the relations of the sciences of physical acoustics, physiology, psychology, and esthetics to the art of music, and more especially, to the theory of harmony "which were received worse than coldly by the heads of the musical profession." However, Mr. Haydon finds that the systematization of harmonic materials and the discussion concerning the treatment of dissonances, and many other features of the Day system should claim our attention and are worthy of the careful consideration of every theorist.

Manfred F. Bukofzer's presentation of the "Evolution of Javanese Tone Systems" may intrigue the hard-headed scientist because of the cataloguing of scales according to vibrational numbers.

Davidson Taylor, of the Columbia Broadcasting System, spoke on "Music Written for the Radio." His paper was largely historical and brought out some interesting factors in the field of "radio-genic" music.

Dayton C. Miller's paper on "Musical Tone-Color (with Phonodeik Demonstrations)" closed the session on Music and Science. It was very briefly presented, touching on the high-lights of his *Science of Musical Sounds* and his *Sound Waves: Their Shape and Speed*. Professor Miller's taste for esthetic values was somewhat naively brought out in a statement that "a beautiful tone has a wave form which is a line of beautiful curves; a rough tone has a jagged wave form which is constantly changing its shape." Another tantalizing notion referred to the effects of temperament. He concludes that "the chord $c^1-e^1-g^1-c^2$, when tuned in pure intonation, produces a wave of beauty which remains unchanged, or is static; the same chord made with tempered intervals produces a wave which is continually changing, as if the spark of life had been given to the sleeping beauty." "While the inanimate beauty may be greater in the abstract, the living beauty is dynamic and more attractive."

Space does not permit of going to the papers other than those given at the session already reviewed, and the scientist may not agree that some of the other papers pertain to a scientific study of music, but even the most hard-headed of scientists would profit at least from the attitudes expressed by the musicologist, and thereby stimulate his genuine appreciation of the stuff from which music is made.

ABE PEPINSKY
Haverford College
Haverford, Pennsylvania

References to Contemporary Papers on Acoustics

MOST of the titles of papers appearing in other languages than English have been translated. Abstracts in English of many foreign papers have appeared or soon will appear in Science Abstracts, Section A. Where references are made to Science Abstracts, the reference is to the volume number and abstract number. The abbreviations of the names of journals are those used in Science Abstracts and can be found in any annual index to those abstracts.

The numbers appearing at the head of each entry are the numbers which identify the subject headings in the Cumulative Index of this Journal issued in November, 1939. The same Classification of Subjects can be found in the index to Volume 16.

Compiled with the generous assistance of B. C. Driver, Physics Librarian, University of Michigan.

2. ARCHITECTURAL ACOUSTICS

2.1 **Sound Waves in Rooms.** PHILIP M. MORSE AND R. H. BOLT. Rev. Mod. Phys. **16**, 69–150 (April, 1944).

2.7 **Sound Waves in Rooms.** PHILIP M. MORSE AND R. H. BOLT. Rev. Mod. Phys. **16**, 69–150 (April, 1944).

4. EAR AND HEARING

4.2 **Cochlear Microphonics. A Critical Review.** P. KELLAWAY. Arch. Otolaryng., Chicago **39**, 203–210 (March, 1944). Sci. Abs. **A8**, 156 (1945).

4.3 **The Effects of Base-in and Base-out Prisms on Stereo-Acuity.** G. A. FRY AND P. R. KENT. Amer. J. Optom. **21**, 492–507 (1944).

4.3 **Psychological Investigations on the Binaural Effect.** J. JOHANNESSON. Arch. ges. Psychol. **111**, 225–287 (1942–1943).

4.3 **They Can See Without Eyes; Facial Perception of Reflected Sound.** A. B. BOUCHER AND J. L. TEHAN. Saturday Evening Post **217**, 26–7 (April 21, 1945).

4.4 **The Relation Between Speech Sound Discrimination and Percentage of Hearing Loss.** J. UTLEY. J. Speech Disorders **9**, 103–113 (1944).

4.4 **Auditory Acuity and Its Relation to Defective Speech.** E. M. SULLIVAN. J. Speech Disorders **9**, 127–130 (1944).

4.4 **Helping the Hard of Hearing Soldier at Deshon.** P. MCCALMONT. Hearing News, Washington **12**, 10–14 (1944).

4.4 For references on Deafness see the Volta Review.

4.4 **The Navy's Program for Rehabilitation of the War Deafened.** F. A. JOSTES. Hearing News, Washington **12**, 3ff (1944).

4.6 **Tonal Audiometric Method for the Group Investigation of the Hearing of School Children.** S. G. NIELSEN. Acta Otolaryng., Stockholm **32**, 263–283 (1944).

4.6 **Audiometer Investigations with Sinusoidal Waves from 16 cycles to 20,000 cycles; the Normal Audiogram and the Audiogram after Investigation of Sodium Salicylate.** A. JUUL. Acta Otolaryng., Stockholm **31**, 386–401 (1943).

4.6 **Differential Sensitivity to Intensity as a Function of the Duration of the Comparison Tone.** W. R. GARNER AND G. A. MILLER. J. exp. Psychol. **34**, 450–463 (1944).

4.9 **Pitch Discrimination as a Function of Tonal Duration.** W. W. TURNBULL. Bull. Canad. psychol. ass. **4**, 77–78 (1944). Abstract.

5. APPLIED ACOUSTICS, INSTRUMENTS AND APPARATUS

5.1 **Aural Surface Comparator (N).** S. M. COX. J. Sci. Instrum. **21**, 219 (Dec. 1944).

5.4 **An Automatic Vibration Analyzer.** F. G. MARBLE. Bell Lab. Record **22**, 376 (April, 1944).

5.16 **The ABC of Photographic Sound Recording.** E. W. KELLOGG. Jour. Soc. Motion Pict. Eng. **44**, 151 (March, 1945).

5.16 **The PH-346A Recording Equipment.** W. C. MILLER. Jour. Soc. Motion Pict. Eng. **44**, 75 (February, 1945).

5.16 **Application of Sound Recording Techniques to Airplane Vibration Analysis.** J. G. FRAYNE AND J. C. DAVIDSON. Jour. Soc. Motion Pict. Eng. **44**, 31 (January, 1945).

5.16 **Aesthetics of Sound Reproduction.** H. A. HARTLEY. Wireless World **50**, 198–202, 236–239 (July and August, 1944); Sci. Abs. **A48**, 157 (1945).

5.16 **Gramophone Needle Buzz.** F. L. DEVEREUX. Wireless World **50**, 290–294 (October, 1944); Sci. Abs. **A48**, 158 (1945).

5.16 **Recordings of Sounds Produced by Certain Disease-Carrying Mosquitoes.** Science **101**, 335–6 (March 30, 1945).

5.16 **Method of Making Cross-Modulation Tests on Variable Area 16 mm Sound Motion Picture Prints.** American Ltd. Assn. **Z52**, 39 (1944).

5.16 **Method of Making Intermodulation Tests on Variable Density 16 mm Sound Motion Picture Prints.** American Ltd. Assn. **Z52**, 15 (1944).

5.16 **Method of Determining Signal-to-Noise Ratio of 16 mm Sound Motion Picture Prints.** American Ltd. Assn. **Z52**, 38 (1945).

6.1 **Certain Applications of Physical Principles to the Playing of Musical Instruments.** W. F. G. SWANN. Franklin Inst. Jour. **239**, 1–26 (January, 1945).

6.1 **Pseudo-Science in Musical 'Theory.'** LLEWELYN S. LLOYD. Proceedings of the Musical Association, Session LXX (March, 1944).

6.1 **Just Intonation Misconceived.** LLEWELYN S. LLOYD. Music and Letters (July, 1943).

6.6 **The Problem of the Keyboard Instrument.** LLEWELYN S. LLOYD. Phil. Mag. [7] **34**, 472 (July, 1943).

6.6 **The Problem of the Keyboard Instrument—II.** LLEWELYN S. LLOYD. Phil. Mag. [7] **34**, 624 (September, 1943).

6.6 **The Problem of the Keyboard Instrument—III.** LLEWELYN S. LLOYD. Phil. Mag. [7] **34**, 674 (October, 1943).

7. NOISE

7.1 **Recordings of Sounds Produced by Certain Disease-Carrying Mosquitoes.** Science **101**, 335–6 (March 30, 1945).

7.1 **Hissing Sounds Heard During the Flight of Fireballs (L).** M. A. R. KHAN. Nature, London **155**, 53 (January 13, 1945).

10. SUPERSONICS (ULTRASONICS)

10.3 **Hand Lapper for Quartz Crystals.** G. M. THURSTON. Bell Lab. Record **22**, 452 (July, 1944).

10.5 **Velocity of Sound and Molecular Association in Liquids.** R. T. LAGEMANN. J. Chem. Phys. **11**, 464–465 (November, 1944).

11. WAVES AND VIBRATIONS

11.3 **Velocity of Sound and Molecular Association in Liquids.** R. T. LAGEMANN. J. Chem. Phys. **11**, 464–465 (November, 1944).

11.3 **Solutions of Problems Relating to Media in Contact by the Method of Wave Trains.** G. GREEN. Phil. Mag. **35**, 519–531 (August, 1944); Sci. Abs. **A48**, 151 (1945).

11.6 **On the Tones of Thin Cylindrical Tubes.** G. E. ALLAN. Phil. Mag. **35**, 571–582 (September, 1944).

11.7 **Studies of Geodesics in Vibrations of Elastic Beams.** A. D. MICHAL. Proc. Nat. Acad. Sci., Washington **31**, 38–43 (January, 1945).

11.7 **Forced Vibrations of Systems with Non-Linear Restoring Force.** K. O. FRIEDRICHS AND J. J. STOKES. Quart. Appl. Math. **1**, 97–115 (July, 1943); Sci. Abs. **A48**, 149 (1945).

11.7 **A Numerical Method on the Theory of Vibrating Bodies.** A. VAZSONYI. J. Appl. Phys. **15**, 598–606 (August, 1944); Sci. Abs. **A48**, 152 (1945).

Review of Acoustical Patents

ROBERT W. YOUNG
C. G. Conn, Ltd., Elkhart, Indiana

Patents reviewed below have been issued by the United States Patent Office on the dates indicated. Printed copies may be obtained from the Commissioner of Patents, Washington, D. C. at a cost of 10¢ each. Remittance may be made by money order, check, cash, or special coupons which are sold in blocks of twenty or books of one hundred.

The decimal numbers appearing before titles are those which identify the headings being used in the analytic subject index of this journal. The Patent Office classification given in parentheses with each patent entry is outlined in the "Manual of Classification of Patents" which is obtainable from the Superintendent of Documents.

The opinions expressed are those of the individual reviewers only.

Reviewers

GEORGE W. DOWNS, *California Institute of Technology, Pasadena 4, California*
C. E. NELSON, *Nelson Muffler Corporation, Stoughton, Wisconsin*
HALE J. SABINE, *The Celotex Corporation, Chicago, Illinois*
L. W. SEPMEYER, *California Institute of Technology, Pasadena 4, California*
FRANK H. SLAYMAKER, *Stromberg Carlson Company, Rochester 3, New York*
R. W. YOUNG, *C. G. Conn, Ltd., Elkhart, Indiana*

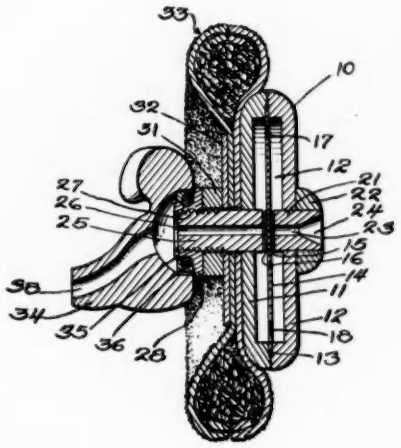
A recent news release from the Office of the Alien Property Custodian directs attention to the fact that abstracts or short descriptions have been prepared for alien-owned U. S. patents. Licenses to most of these patents are available to U. S. citizens at a nominal fee of $15.00 per patent and are good for the life of the patent. An index, showing the section or class titles and prices of the abstracts, may be obtained free of charge by writing to the Alien Property Custodian, 311 Field Building, Chicago 3, Illinois.

2,361,963

4.5 ACOUSTIC GUARD

Maurice C. Rosenblatt.
November 7, 1944, 18 Claims (Cl. 181–22).

This acoustic guard, according to the inventor, provides an acoustic peak chopper to reduce the intensity of extremely loud sounds. Diaphragm **12** is fitted with pads **15** and **16** which are closely spaced to ports **23** and **26.** At **17** and **18** are bypasses which permit the passage of normal sounds up to approximately 85 db intensity level. Above this level the diaphragm vibrates with an amplitude sufficient to close first one and then the other of the ports **23, 26,** thus restricting the passage of sound. Such a device,



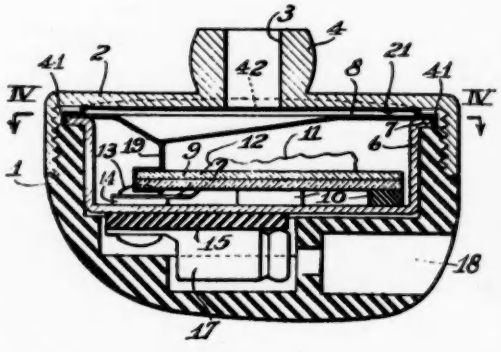
it is stated, would enable a soldier to hear commands while at the same time excluding the more intense sound of battle.—LWS

2,367,726

4.5 SOUND TRANSMITTER

Samuel F. Lybarger, assignor to E. A. Myers & Sons.
January 23, 1945, 6 Claims (Cl. 181–32).

The diaphragm **8** in this hearing aid type receiver is driven off center at the apex of a cone shape depression formed in the diaphragm. By using the off-center drive, the inventor has been able to use a considerably larger

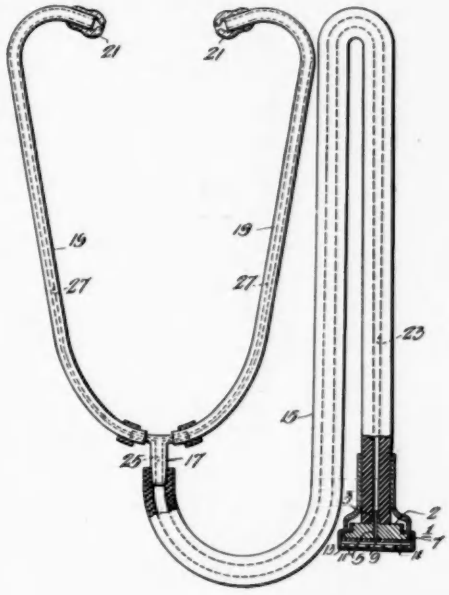


Rochelle salt crystal **9** than would normally be used in a small case. To facilitate renewal of the crystal unit should it be damaged, the unit is housed in a removable metal cup **6.**—FHS

2,363,686
5.1 ACOUSTIC STETHOSCOPE

Harry F. Olson, assignor to Radio Corporation of America.
November 28, 1944, 20 Claims (Cl. 181–24).

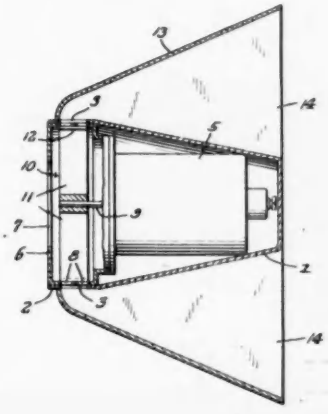
Relatively uniform frequency response in an acoustic stethoscope is maintained from about 30 to 4000 cycles/sec.



by a novel design of the cavity back of the diaphragm **13** and an impedance matching tapered passage **23.** Good low frequency response is obtained by the use of a limp diaphragm. A multitude of resilient projections **11** support the diaphragm, so that it does not deflect to close the opening **9,** yet the cavity is small so the high frequency response is good.—RWY

2,367,233
5.1 SIREN

Frederick C. Millard.
January 16, 1945, 1 Claim (Cl. 177–7).

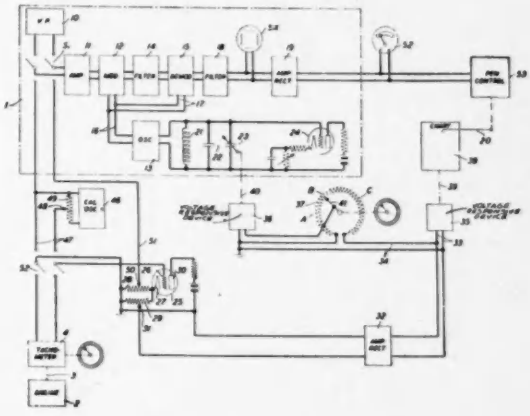


An essential feature of this siren is the fact that the incoming air (at **7**) and outgoing air (at **14**) both flow in the same general direction.—RWY

2,361,990
5.4 VIBRATION ANALYZER

William J. Brown, assignor to Western Electric Company, Incorporated.
November 7, 1944, 9 Claims (Cl. 234–5.6).

A feature which distinguishes this vibration analyzer is its ability to follow the changes in frequency of a device under test, such as an airplane engine, and to record the amplitude of some vibration component whose frequency is proportional to the engine speed. The proportionality can be selected at will so it is possible by successive sweeps of engine speed to trace out the spectrum of the sound as a function of speed with as much detail as may be desired.

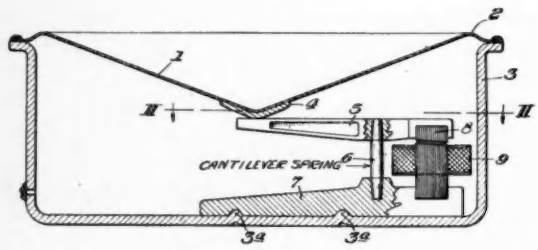


A tachometer **4** is used whose output voltage and frequency are proportional to engine speed. Through device **36,** this voltage adjusts automatically the pass frequency of heterodyne analyzer **1** and the abscissa position of chart **38.** Contact **B** on potentiometer **37** may be in a position such that the analyzer is adjusted to the fundamental rotational speed of the engine. When the contact is at **C** the component being recorded will be twice the rotational speed, etc. The analyzer may be operated by an airplane test pilot without a helper.—RWY

2,364,485
5.8 ACOUSTICAL DEVICE

Charles B. Spencer.
December 5, 1944, 8 Claims (Cl. 177–7).

Anyone who has done quantitative work with mechanical or acoustical vibrating systems will find in this patent a number of familiar and amusing fallacies. The device, which is intended to be useful either as an auto horn or a loudspeaker, encloses an evacuated chamber back of the metal diaphragm **1,** so the diaphragm, as the patent states, "does not waste any power working the atmosphere on its back



side." The spring **6** and lever **5** are to be strong enough to withstand the terrific atmospheric pressure on the diaphragm, yet the patent further states that by proper design of this relatively heavy die cast lever and the stiff cantilever spring, the resonance point of the system can be moved out of the audible range. The distortion introduced by the non-linear iron armature driving motor, the inventor minimizes by proportioning the lever arms properly. The distortion is, supposedly, further reduced by the fact that as the iron armature moves away from the magnet core, both the driving force and the opposing force caused by the spring become less. As an auto horn, however, the diaphragm vibrates at double the exciting frequency.—FHS

2,353,089
5.9 TRANSDUCER

Harry B. Shapiro, assignor to the Brush Development Company.
July 4, 1944, 18 Claims (Cl. 171–327).

The inventor has devised a cartridge for a piezoelectric unit which can be hermetically sealed and yet which is

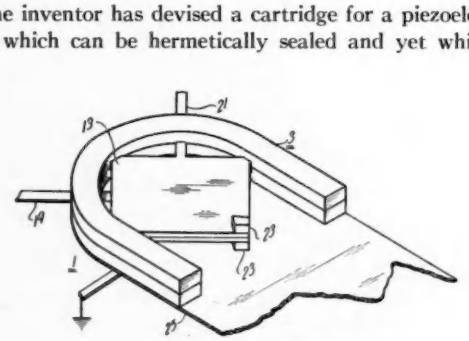


FIG. 1.

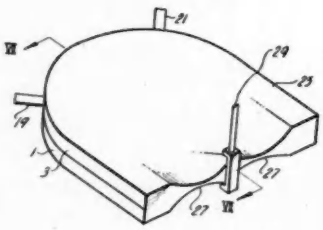


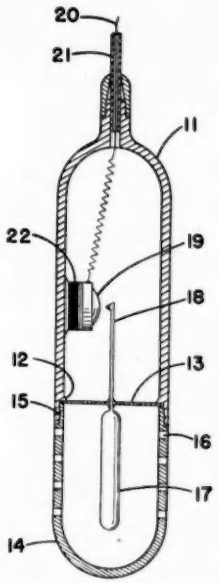
FIG. 2.

intended to permit normal action of the crystal. In Fig. 1, the U shaped frame **3** holds a twister type Bimorph crystal **13** at three corners. Blocks **23** of Bakelite, Celluloid, or similar material mounted on the crystal at the free corner transmit the vibration from the drive link **29** in Fig. 2 to the crystal. The flexible moisture proof cover sheet **25** is folded over the entire assembly.—FHS

2,361,458
5.9 MICROPHONE FOR CONDUITS

Lynn C. Converse, assignor to Standard Oil Development Company.
October 31, 1944, 3 Claims (Cl. 177–311).

The microphone described in this patent is intended for use in hunting leaks in oil well casings. When the openings **16** in the microphone case are opposite either a leak in the

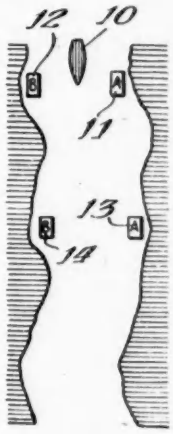


casing or a point where water is entering, the vibrations thus produced actuate the microphone unit **19** through vane **17** and rod **18.** A flexible diaphragm **13** acts as a support for the vane and also prevents the entry of any fluid into the microphone chamber. The microphone is intended to respond to vibrations traveling across the case but not to vibrations traveling longitudinally.—FHS

2,366,800
5.10 INDICATING DEVICE

Ernst Norrman.
January 9, 1945, 6 Claims (Cl. 177–352).

A specific application of this radio-acoustic-ranging device is to the navigation of a ship between two points such as the buoys **11** and **12.** Each buoy is equipped with a sound transmitter under the control of a radio receiver. When an appropriate radio signal is sent out from ship **10,** sound is emitted at each buoy (but at differing frequencies). The difference in the times of arrival of these sounds

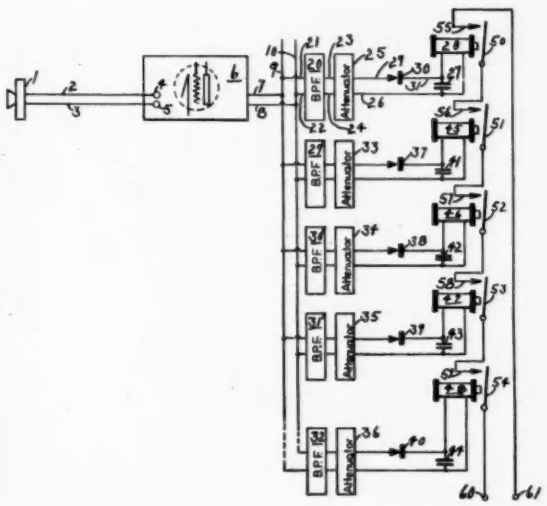


actuates a device disclosed in the patent in such a manner as to show to which buoy the ship may be nearer. By keeping the ship equidistant from a succession of such buoys it may thus be kept in the middle of the channel. —RWY

2,368,953
5.10 ELECTRIC CONTROL SYSTEM

Phillip John Walsh.
February 6, 1945, 3 Claims (Cl. 177–352).

This invention is described in terms of a protective device against torpedoes. The underwater sound of the torpedo is picked up by hydrophone **1** and in effect is analyzed by band pass filters **20, 29, 30, 31, 32.** *If* the sound

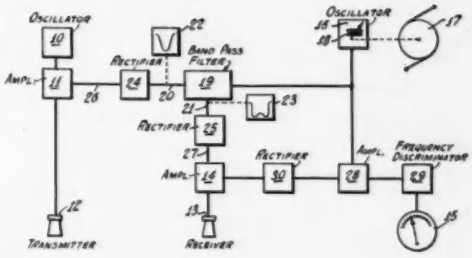


spectrum is that for which these filters and attenuators **25, 33, 34, 35, 36** have been adjusted, then the associated relays will close and the circuit between terminals **60** and **61** will be completed and capable of initiating automatically countermeasures against the torpedo. Methods are discussed for balancing out the noise of the ship on which the device is installed.—RWY

2,371,988
5.10 DISTANCE MEASURING DEVICE

Carl-Erik Granqvist, assignor to Aga-Baltic Aktiebolag.
March 20, 1945, 11 Claims (Cl. 177–352).

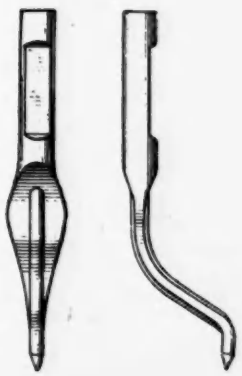
In this echo-sounding device the frequency from motor-driven oscillator **16** existing at the time the echo is received is used as a measure of the distance to the echo-producing surface. The transmission loss of the band pass filter **19** in the two directions is indicated at **22** and **23** respectively. As the frequency of the oscillator output increases it first causes rectifier **25** to render receiver **13** inoperative. Then as the frequency becomes slightly greater the signal passed



to rectifier **24** causes amplifier **11** to send out a pulse of supersonic energy through transmitter **12.** When the reflected pulse is received it causes rectifier **30** to admit the oscillator signal to frequency discriminator **29.** Since the oscillator frequency changes linearly with time the frequency at the moment the pulse is received is a measure of the distance to the reflecting surface, which distance may be read directly on scale **15.**—RWY

Des. 139,584
5.16 DESIGN FOR A PHONOGRAPH NEEDLE

Peter L. Jensen.
November 28, 1944 (Cl. D26–14).

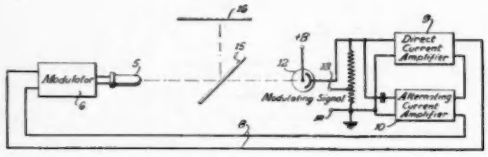


2,360,012

5.16 LAMP MODULATED RECORDING METHOD AND SYSTEM

Hillel I. Reiskind, assignor to Radio Corporation of America.
October 10, 1944, 10 Claims (Cl. 179–100.3).

By employing in a lamp modulated recording system both d.c. and a.c. feedback, including the optical path in conjunction with the recording lamp, the average light intensity is maintained constant, and the frequency and



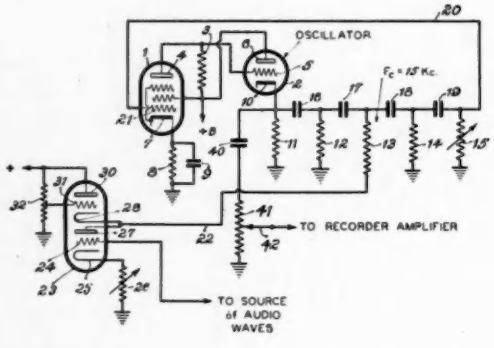
amplitude distortions are held to a low value. This is accomplished by interposing the partially reflecting mirror between lamp **5** and film **16** and having the transmitted light fall on photoelectric cell **12**.—LWS

2,361,658

5.16 SOUND RECORDING AND REPRODUCING SYSTEM

Chester M. Sinnett, assignor to Radio Corporation of America.
October 31, 1944, 6 Claims (Cl. 179–100.1).

A resistance capacitance phase shift oscillator is used to produce a frequency modulated carrier for use in recording



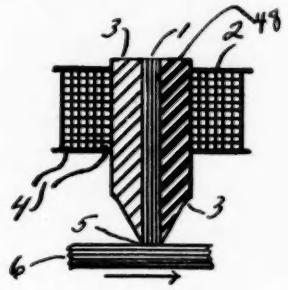
according to a method disclosed by W. van B. Roberts in application Serial Number 369,829.—LWS

2,361,753

5.16 MAGNETIC POLE PIECE

Stanley D. Eilenberger, assignor to Wolgen Company.
October 31, 1944, 11 Claims (Cl. 179–100.2).

In order to afford resistance to wear this pole tip for use in magnetic recording or reproduction is furnished with a non-magnetic sheath. It is stated that this sheath, if made of material having good electric conductivity, will constitute an eddy current shield and hence prevent the spreading of the pole tip flux. On the other hand, the sheath is



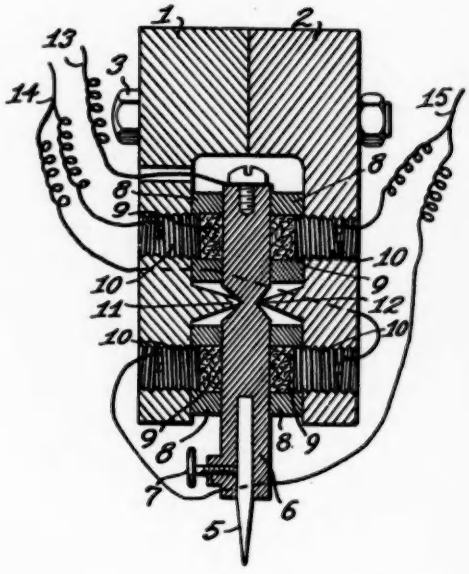
provided with an insulated slot to prevent the formation of a shorted turn about the pole. The use of and a method for forming plasticized powdered iron pole tips are claimed. —LWS

2,368,003

5.16 MECHANICAL-ELECTRICAL SOUND REPRODUCER

Georges Courcy and Marcel Paul Durand; vested in the Alien Property Custodian.
January 23, 1945, 3 Claims (Cl. 179–100.41).

Resistance elements such as carbon buttons are employed to modulate a polarizing current in accordance with



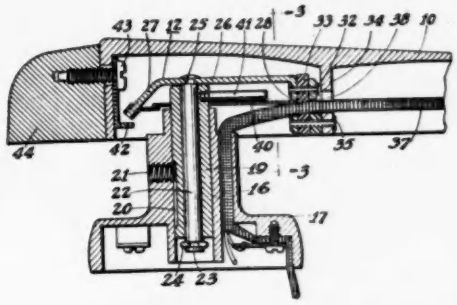
the undulations of a record groove. Methods of using one, two or four buttons are disclosed.—LWS

2,368,198

5.16 CONNECTION MEANS

Walter J. Brown and Alfred L. W. Williams, assignors to The Brush Development Company.
January 30, 1945, 6 Claims (Cl. 274–1).

An object of this invention is to eliminate the distortion and tone arm resonance in phonograph reproducers caused



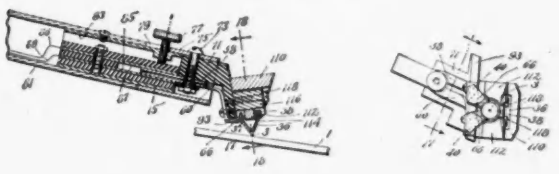
by loose bearings. This is accomplished primarily by substituting a hinge **33** of resilient material for the usual axle-type bearing.—LWS

2,369,676

5.16 ELECTROMECHANICAL-CONVERSION DEVICE

Frederick Vinton Hunt and John Alvin Pierce.
February 20, 1945, 35 Claims (Cl. 179–100.41).

This patent relates to improvements on the design of a phonograph reproducer previously disclosed in Patent 2,239,717. The design is unique in that the ribbon-like electrical conductor also forms the vibratory system, the



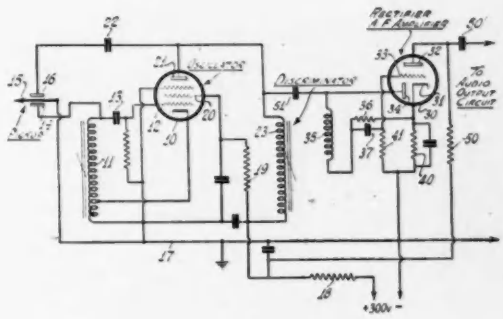
characteristics of the latter being controlled by the twist imparted to the ribbon. The patent teaches the mathematical relationship between the various factors entering in the design. The coin silver conductors **66** serve also to support the stylus carrying cone **36.**—LWS

2,371,373

5.16 BALANCED FREQUENCY MODULATION SYSTEM

Alexis Badmaieff, assignor to Radio Corporation of America.
March 13, 1945, 14 Claims (Cl. 179–100.4).

This FM system of reproducing phonograph records provides a balanced or push-pull pick-up in which one



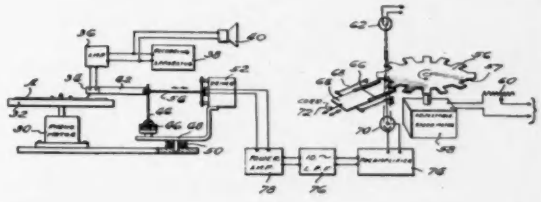
condenser plate varies the frequency of the oscillator while the other varies the tuning of the discriminator, the two actions being out of phase with each other by 180°. It is stated that this results in a doubled output and a reduction in harmonic generation.—LWS

2,373,560

5.16 SOUND RECORDING METHOD AND APPARATUS

John M. Hanert, assignor to Hammond Instrument Company.
April 10, 1945, 29 Claims (Cl. 179–100.4).

This invention makes it possible to impart a vibrato to the recorded sound of a musical instrument, such as a piano, on which ordinarily the vibrato is not available. The usual recording system is indicated by **38, 40** being a monitor speaker. The cutter **34** is supported in such a manner that it may be moved at a suitable vibrato rate



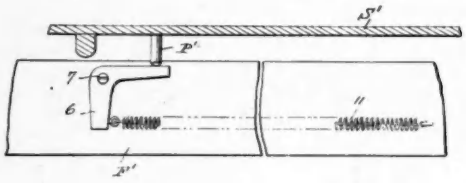
such as seven per second. One method of deriving this vibrato frequency is indicated by the light-chopper arrangement **56,** which becomes operative when the shutter **64** is pulled back to admit the light from **62** to the phototube **70.**—RWY

Re. 22,578

6.6 PIANO

Archie Mitchell Tyson.
December 12, 1944, 4 Claims (Cl. 84–189).

An object of this attachment for a piano is to produce a "singing" quality as by the continuing vibration of spring



11. This receives its motion through bell-crank **6** from the sound board S′.—RWY

Des. 139,861

6.7 DESIGN FOR A VIOLIN OR SIMILAR ARTICLE

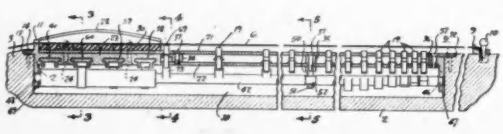
Herbert Lee Miller.
December 26, 1944 (Cl. D56–1).

2,368,256

6.7 STRINGED MUSICAL INSTRUMENT

John W. McBride.
January 30, 1945, 17 Claims (Cl. 84–315).

This invention is an improvement over those disclosed in prior applications (by the same inventor) wherein "there are described mechanisms adapted to be operated by the fingers of a player, for determining the effective length of any of the strings, and thereby to determine the pitch of the sounds produced by the vibrating strings. The important part of the mechanisms therein described is a rod for each string, and rotatable about an axis having generally the same direction as the string. This rod carries either a continuous or interrupted helical-like crest or its equivalent. The crest is adapted to contact the string directly; and the point of contact is determined by the angular position of the rod." Here the crests on the rods elevate frets at the proper positions. If the movement is sufficiently rapid the string may be set in motion by the contact of the fret.—RWY

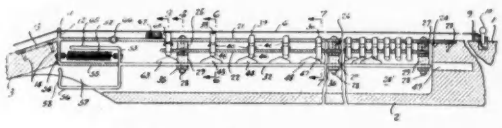


2,368,257

6.7 STRINGED INSTRUMENT

John W. McBride.
January 30, 1945, 10 Claims (Cl. 84–315).

In place of a rotary motion (see preceding patent, also J. Acous. Soc. Am. **15,** 206–207 (1944)) in this arrangement the inventor stops the strings of a musical instrument by



frets such as **38, 39** which are moved against the string by cams **43, 44.** The operating rod **32** is moved parallel to the string by "manually manipulable member 56."—RWY

Des. 139,332

6.8 DESIGN FOR A FLAGEOLET

Josephus Thompson, assignor of one-third to Grossman Music Company.
October 31, 1944 (Cl. D56–1).

Des. 139,333

6.8 DESIGN FOR A FLUTE

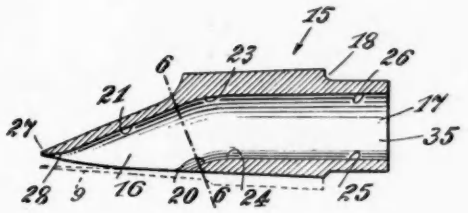
Josephus Thompson, assignor of one-third to Grossman Music Company.
October 31, 1944 (Cl. D56–1).

2,368,556

6.8 MOUTHPIECE FOR REED INSTRUMENTS

Mario Maccaferri.
January 30, 1945, 1 Claim (Cl. 84–383).

A notable feature of this mouthpiece for a reed instrument, apparently a saxophone, is claimed to be the "highly polished smooth interior channel with a surface free from crevices, dents or shoulders and of substantially uniform cross-sectional area from the reed end to the tube end. . . ." That the end of the mouthpipe projecting



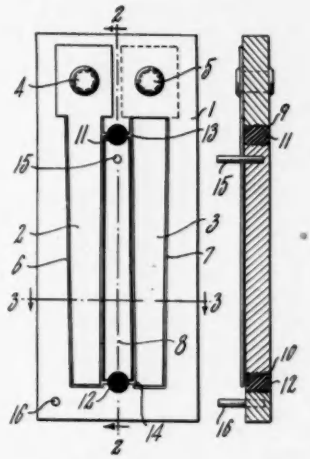
into the mouthpiece creates a sizeable shoulder seems to be ignored. The explanation given of the supposed vibration of the air within the instrument oscillates between the assumption of an "open" and "closed" end at the mouthpiece.—RWY

2,368,842

6.9 MUSICAL INSTRUMENT

Ylan K. Kealoha.
February 6, 1945, 7 Claims (Cl. 84–1.14).

This accordion reed plate is so constructed that the portion **8** is electrically insulated from the vibrating reeds **2, 3.** The inserts of insulating material are **11, 12.** The



variable capacitance between the parts insulated from each other may be utilized in an electrostatic pick-up for electrical amplification. The total weight of the reed plate is little changed by this modification.—RWY

Program of the Thirtieth Meeting of the Acoustical Society of America

HOTEL PENNSYLVANIA, NEW YORK, NEW YORK

May 11–12, 1945

1. The Vibratron. FRANK RIEBER, *Rieber Research Laboratory, New York, New York* (15 min.).—A continuously adjustable high Q electromechanical resonator, which may be used in the control of oscillators, pass-band amplifiers, and the like. Applications of the device to laboratory apparatus, and to the telemetering and signaling fields, are described.

2. Transmission, Reflection, and Guiding of an Exponential Pulse by a Steel Plate in Water. M. F. M. OSBORNE AND S. D. HART, *Sound Division, Naval Research Laboratory, Washington, D. C.* (15 min.).—The problem of the action of an underwater explosion—a pulse of the form $p_{max}e^{-t/\tau}$—on an elastic plate in water can be divided into two parts. In the first, the plate is considered as an infinite slab with the explosion wave as an incident plane wave transient. In the second, the plate is considered as a wave guide, with the explosion wave setting the initial conditions at the edge of the plate. The first is an inhomogeneous boundary value problem, the second a homogeneous one. The inhomogeneous case has been solved in the literature, for continuous waves, and the results of the theory are in qualitative agreement with the experiments on transients. The homogeneous problem forms the principal part of this paper. The phase velocity curves, as functions of frequency, for the principal normal modes of a plate in water, are closely related to the corresponding curves for a plate in a vacuum. The effect of the water on the principal symmetric mode of the plate is not to change the real part of the phase velocity from its value for a plate in a vacuum, but to add a small imaginary component, or attenuation due to radiation loss into the water. The principal antisymmetric mode is slightly attenuated at high frequencies, and strongly attenuated at low frequencies. The cut-off comes at that frequency where the phase velocity along the plate equals the velocity of sound in water. In addition to these two modes, which correspond closely to the principal modes for a plate in a vacuum, the presence of the water introduces two additional modes, one symmetric and one antisymmetric. At high frequencies, their phase velocities are very close to, but a constant fraction of, the velocity of sound in water. At low frequencies, the phase velocity of the symmetric mode approaches the velocity of sound in water, the phase velocity of the antisymmetric mode approaches zero. Both these modes are unattenuated, i.e., their phase velocities are all real. The problem of the propagation of a transient in a wave guide presents the interesting mathematical problem, as yet unsolved rigorously, of the contour integral of a function of a complex variable defined implicitly. Brief comparison of the theory with experiment shows that the pure real antisymmetric mode accounts for most of the properties of the precursor. This is an oscillation which precedes the explosion wave along the plate, at a velocity slightly greater than that of sound in water, but less than that of rotational or dilational waves in the plate.

3. The Driving-Point Impedance of an Infinite Solid Plate. R. CLARK JONES,* *Polaroid Corporation, Cambridge, Massachusetts* (15 min.).—In the design of mechanical filters whose function it is to prevent the transmission of vibration from one structure to another, it is necessary to know the impedances of the structures between which the filter is to be connected. In many of the cases which arise in practice, the impedance may be estimated from a knowledge of the driving-point impedance of an infinite plate. An equation is presented for the impedance of an infinite plate of constant thickness when connection is made to the plate at a single point. The impedance may be written in the form $8\rho h^2 v$, where ρ is the density, h the thickness of the plate, and v a velocity approximately equal to the velocity of shear waves in the material. It is shown that the driving point impedance is equal, except for a constant factor, to the impedance of a mass equal to the mass of a disk cut from the plate whose radius is the mean proportional of the thickness and the wave-length which corresponds to v. The results are exemplified by applying them to the case of a steel plate.

* The work reported here was carried out when the writer was a Member of the Technical Staff of the Bell Telephone Laboratories.

4. Absorption and Scattering by Absorbent Cylinders. RICHARD K. COOK, *National Bureau of Standards, Washington, D. C.* (10 min.).—The absorption and scattering of a plane wave by an infinitely long circular cylinder, whose axis is perpendicular to the direction of propagation of the wave, are calculated. The surface of the cylinder is assumed to have a known normal acoustic impedance. Absorption measurements made on long cylinders placed in a reverberation room (where the incident wave directions are at all angles to the axes of the cylinders) are compared with the computed values.

5. Theory of the Effect of Wall Irregularities on the Distribution of Sound in a Room. P. M. MORSE, *Massachusetts Institute of Technology, Cambridge, Massachusetts* (20 min.).—The perturbation theory of Feshback and Clogston can be applied to determine the amount of wall irregularity required to produce adequate "randomness" in the sound waves in a room. When the second-order terms in the perturbation series are as large as the first-order terms, then each standing wave is a complete mixture of all the types of simple waves, and the decay rate approaches that predicted by the Sabine formula.

From this criterion one can obtain an "index of randomness," which is a measure of the ability of the room to "mix" sound of a given frequency adequately. If the index is much smaller than unity, the sound will not be well mixed, and the decay curve will be a curved line (Sabine formula will not hold). If the index is much larger than unity, the Sabine formula will hold. Approximate formulas for the index show that irregularies in wall shape are usually more effective (in mixing the sound) than irregular patches of absorbing material, that patches (or bumps) of the order of size of a half-wave-length are more effective than larger or smaller ones, and that rooms large compared to the wave-length are easier to "randomize" than smaller rooms. It is quite difficult to treat a room of dimensions smaller than ten wave-lengths so that the index of randomness comes out larger than unity.

6. Influence of Room Proportions on Normal Frequency Spacing. RICHARD H. BOLT, *Massachusetts Institute of Technology, Cambridge, Massachusetts* (15 min.).—Proportioning of rectangular rooms to give smooth low frequency response has been guided by an empirical rule: dimensions should be related by ratios of $2^{\frac{1}{3}}$ or its powers. An analytical basis for a criterion is being explored. Normal frequencies and their distributions are expressed in generalized terms, using dimensions L, pL, qL, and a dimensionless frequency parameter μ. Distributions of μ can be investigated as a function of proportions (p, q) alone, holding volume constant. A *frequency spacing index* Ψ is obtained by averaging the ratio of *actual* to *average* spaces, weighted by the width of each space. The index Ψ increases with increasing fluctuation from uniform spacing; $\Psi=1$ for uniform spacing; $\Psi=2$ for random (Poisson) distribution. The index has been evaluated numerically over arbitrary frequency intervals, for a number of room proportions. As expected, Ψ is large (>2) for a cube and other proportions with simple ratios. The index has a small value (<1.5) over ranges of incommensurate p, q values. The "$2^{\frac{1}{3}}$ rule" gives results in the low region, but is apparently not a unique measure of optimum smoothness. These calculations are approximately correct for basically rectangular rooms having irregularities (e.g., polycylindrical diffusers) appreciably smaller than wave-lengths involved. Room proportions, and the Ψ-criterion, are probably not important when room dimensions are longer than about 10 wave-lengths, as adequate diffusion can then be obtained by shape irregularities and absorption.

7. Detection of Resonances in Vibratile Members by Measurement of Electro-Magnetically Induced Voltages. R. W. CARLISLE AND H. W. KOREN, *Sonotone Corporation, Elmsford, New York* (15 min.).—An instrument is described by which the velocity *versus* frequency characteristics of a conducting filament or similar vibratile member may be detected, regardless of the inaccessibility of the member. The assembly to be studied is submerged in a known magnetic field and subjected to vibration over the audio range. The voltage generated in the vibratile member is amplified and this indicates the integrated vibration velocity of the member.

8. Headphone Measurements and Their Interpretation. DANIEL W. MARTIN AND LESLIE J. ANDERSON, *RCA Victor Division, Indianapolis, Indiana* (15 min.).—Methods of acoustical measurement and of data presentation are reviewed, with emphasis on relating headphone data to the requirements of the communication and monitoring fields of application. The conditions of headphone listening are compared to standard and typical listening conditions, as an aid to the interpretation of headphone measurements. Data are shown to demonstrate the importance of statistical analyses of the performance of headphones on different wearers. Some effects of presenting too "artificial" an acoustical load to the receiver during measurement are illustrated. An experiment in artificial ear development is described. The requirements for an improved artificial ear are outlined, based in part upon the experimental work.

9. Basic Measurements on the Physical Characteristics of Speech Transmission Systems. L. L. BERANEK, *Electro-Acoustic Laboratory, Harvard University, Cambridge, Massachusetts* (15 min.).—This paper discusses proposed American standardization of basic tests on the physical characteristics of speech transmission systems.

10. Acoustic Pulse Testing. E. F. SHRADER,* *Columbia University, New York, New York* (15 min.).—In the testing of electro-acoustic transducers such as microphones one desires to find the generated electrical signal in the presence of an incident plane wave sound field. Likewise, in testing loudspeakers, one wishes to measure the generated sound field. In practical testing systems, reflections which interfere with the measurement are in general present even in the best of "dead" rooms. The pulse testing procedure has been successful in acoustic measurements in eliminating the effects of reflections from the boundaries of the medium. The method consists of generating in the medium a short finite train of sinusoidal waves whose wave-length corresponds to the frequency at which the measurement is desired. The directly transmitted pulse is measured before any reflected signal reaches the sound pickup. The reverberation level is allowed to become negligible before the next pulse is generated and measured. Electronic circuits for generating the applied electrical signal and for continuously recording the received signal level will be described. The system also has been designed so that the direct signal may be discriminated against and any reflection measured, thus providing a means of measuring reflection coefficients of acoustic materials. The limitations of the pulse method will be discussed and compared with those of frequency warble and noise methods of vitiating reflections.

* On leave from Case School of Applied Science.

11. Schlieren Photography of Sound Waves. F. D. CARLSON, K. C. CLARK AND J. C. EISENSTEIN, *Electro-Acoustic Laboratory, Harvard University, Cambridge, Massachusetts* (15 min.).—This paper summarizes the historical background of the schlieren detection technique and gives a qualitative and quantitative description of its

operation. The optical and electronic features of a working schlieren apparatus with high speed illumination are described. The equipment provides visual or photographic observation of an acoustic wave, conveniently generated by a spark gap, at a precisely adjustable time after its formation. Typical measurements of photographic density across wave fronts are shown and discussed. Applications of the schlieren technique in certain acoustical problems are mentioned, including the reflection, diffraction and refraction of sound and the variation of propagation velocity and wave form in sound waves of finite amplitude.

12. A Working Standard for Sound Pressure Measurements. FRANK MASSA, *Electro-Acoustic Consultant, Cleveland Heights, Ohio* (15 min.).—This paper discusses the necessary requirements to be met in an ideal sound pressure measurement standard. It also describes the design and shows the characteristics of a new microphone that meets these requirements to a greater degree than do existing units which are generally available for use as laboratory standards.

13. A General Proof of the Reciprocity Theorem for Electroacoustic Transducers. LESLIE L. FOLDY AND HENRY PRIMAKOFF,† *Columbia University, New York, New York* (15 min.).—On the basis of a general theory of the operation of passive linear transducers developed by the authors, a proof of the reciprocity theorem for electroacoustic transducers†† is presented together with the conditions necessary for its validity. The reciprocity theorem states that the ratio of the microphone response to the speaker response for an electroacoustic transducer is a quantity which, under certain general conditions, is independent of the specific nature and construction of the transducer. This quantity is $2d/\rho f$ where ρ is the density of the medium, f is the frequency, and d is the distance from the center of the transducer to the point where the pressure is measured in obtaining the speaker response. The center of the transducer is defined as the point where the incident pressure is measured, in the absence of the transducer, when obtaining the microphone response. The general theory referred to above assumes a pair of the most general equations (in this case, integral equations), relating the pressure and normal velocity at each point on the surface of the transducer and the voltage and current at its terminals. These, together with the appropriate solutions of the wave equation derived by the use of Green's functions and relating the pressure at any point in the medium to the normal velocity distribution on the transducer surface, completely characterize the behavior of the transducer and allow explicit calculation of the responses. The validity of the reciprocity theorem then requires certain symmetry conditions on the parameters entering into the equations. The relation between these symmetry conditions and the physical characteristics of the transducer allow the limits of validity of the reciprocity theorem to be delineated.

† On leave from Queens College.
†† Schottky, Zeits f. Physik **36**, 689 (1926). MacLean, J. Acous. Soc. Am. **12**, 140 (1940).

14. The Absolute Calibration of Condenser Microphones by the Reciprocity Method. A Review. FRANCIS M. WIENER, *Electro-Acoustic Laboratory, Harvard University, Cambridge, Massachusetts* (15 min.).—The reciprocity method of securing absolute calibrations of condenser microphones offers several important advantages over the thermophone and other methods, which it has effectively displaced in recent years. Both pressure and free-field calibrations† can be obtained by this method. Through cooperation between the Bell Telephone Laboratories, the National Bureau of Standards, and the Electro-Acoustic Laboratory, Harvard University, the technique of obtaining absolute pressure calibrations of condenser microphones by the reciprocity method has been perfected and standardized to a gratifying degree of accuracy. Work on absolute free-field calibrations has, in general, progressed at a somewhat slower rate. A review of the work of the past several years is given together with a discussion of some of the problems encountered.

† American Recommended Practice for the Calibration of Microphones, American Standards Association, Z24.4–1938.

15. Measurement of Acoustic Impedance by the Room Transmission-Characteristic Method. CYRIL M. HARRIS, *Massachusetts Institute of Technology, Cambridge, Massachusetts* (15 min.).—The value of the decay constant and frequency of a normal mode of vibration of a rectangular room with uniform boundaries of known impedance can be predicted from the wave theory of room acoustics. Conversely, from measurements of these quantities from the room's transmission-characteristic, i.e., from the width of the resonance peak and the frequency of the peak's maximum, the boundary impedance may be determined. This value of the impedance is checked by comparing the pressure distribution for the mode obtained theoretically (with the use of the determined impedance value) with the pressure distribution experimentally obtained. With this method, one can determine the acoustic impedance at all angles of incidence, at very low frequencies, for large samples, and under actual mounting conditions.

16. Frequency Response Preference Tests for Record Reproduction. B. B. BAUER, *Shure Brothers, Chicago, Illinois* (15 min.).—The design of a sound-reproducing system often involves a number of compromises which can be resolved only by resorting to listening tests. Some of the principles involved in administering and interpreting such tests are considered. Equipment designed to test frequency-response preference of listeners for recorded music is described. Tests given to technical and to non-technical audiences showed that the former give preference to a more extended frequency range than the latter, despite increased audibility of surface noise and distortion. These factors must be kept in mind by the engineers who design equipment for use by the general public.

17. Practical Considerations of "Higher Fidelity" in Sound Transmission and Reproduction. GEORGE M. NIXON, *National Broadcasting Company, New York, New York* (20 min.).—The paper discusses the various factors which influence the performance of a sound system and

which must be considered in the provision of a "higher fidelity" service. The effects of loudness, noise, hearing, sound absorption etc., are included in the discussion.

18. The Problem of Frequency Range in Speech and Music Reproduction. RALPH P. GLOVER, *Consulting Engineer, Chicago, Illinois* (15 min.).—The specification of a frequency range for a speech or music reproducing system is a complicated matter, with immediate implications as to quality, efficiency, cost, and bulk. The boundaries of the spectra of speech and music are well established but the cost of reproducing these extreme ranges in a near-ideal manner is prohibitive for all ordinary applications. Studies of masking by noise and of hearing ability in the population indicate that statistically there is no real loss in restricting the band width to the order of half that of the original spectrum. Furthermore, it is the opinion of many experienced observers that something less than the complete spectrum yields optimum listening enjoyment and satisfaction. Further restriction of the frequency range entails loss of quality for essentially distortion-free sources with low noise content. The situation is examined for both speech and music with the aid of frequency diagrams for the audio frequency range which show the relationships between cut-off frequencies, band widths and the various criteria of quality and intelligibility. A rational system for departing from the complete frequency range can be developed in the form of a preferred series of reproduction bands based on the liminal intervals of Gannett and Kearney and aural balance relationships. This provides a practical approach to a system of bands with definite but just-perceptible quality differences between them, which is a most useful concept and one which would greatly aid in the engineering of sound systems. In the case of amplified speech communication, where the emphasis shifts from quality to effectiveness from a communications viewpoint, articulation, power economy, directivity and similar considerations, different bands are necessary and possible frequency ranges are discussed.

19. Factors Governing the Intelligibility of Speech Sounds. N. R. FRENCH AND J. C. STEINBERG, *Bell Telephone Laboratories* (20 min.).—This paper discusses the factors which govern the intelligibility of speech sounds and presents relationships which have been developed for expressing quantitatively, in terms of the fundamental characteristics of speech and hearing, the capability of the ear in recognizing these sounds. The success of the listener in recognizing and interpreting speech sounds depends on their intensity in the ear and the intensity of unwanted sounds that may be present. The relationships presented in this paper deal with the intelligibility of speech sounds as a function of these intensities in the different frequency ranges of speech. Results calculated from these relationships are compared with observed results for a variety of systems.

20. Relation Between the Theory of Hearing and the Interpretation of Speech Sounds. W. A. MUNSON, *Bell Telephone Laboratories* (20 min.).—The theory that the interpretation of speech sounds is largely dependent upon differential sensitivity, i.e., the ability of the ear to detect small changes of pitch and intensity, is used as a basis for predicting the results of intelligibility tests. By use of empirical methods, a functional relation is obtained between the energy distribution of speech, the differential sensitivity of the ear, the masking properties of sounds, and the number of correctly perceived syllables in an articulation test. A comparison is made of the empirical functions and the corresponding fundamental speech and hearing characteristics. It was found that in most instances computed intelligibility compares favorably with experimental results, but in a few cases large discrepancies were noted.

21. The Place of Music in Healing. HAROLD BURRIS-MEYER AND R. L. CARDINELL (15 min.).—The current vogue for employing music in hospitals, convalescent and rehabilitation centers, and the publicity attendant thereon, has bred a popular belief that music is a well-understood and essential element in therapy. This paper undertakes to survey and evaluate current practice, to define the scope of what can safely be undertaken in the light of existing knowledge, and to outline an analytical approach to the problem of finding out what music can do toward the alleviation of human ills. Studies in the effect of music on men, the control of emotion by auditory means, and music in industry are cited as examples of techniques and indications of the direction in which progress may be anticipated.

22. The Musical Scale of Just Intonation. NOEL URQUHART, *Control Instrument Company, Inc., Brooklyn, New York* (15 min.).—The musical scale of Just Intonation in one tonality (key) as evolved since early Greek times, may be defined as consisting of four tonal notes: subdominant, tonic, dominant and supertonic at intervals of fifths (3/2) apart with four major notes, each an interval of the major third (5/4) above these tonal notes and with four minor notes, each an interval of the major third (5/4) below these tonal notes, all brought within the limits of an octave (2/1). Assuming the frequency of the keynote of *C* equals 260.74 cycles per second, we obtain the following scale of frequencies:

Note	Name of Interval	Frequency Ratio to Prime		Frequency Cycles/Second
C	Prime (Tonic)	1/1	1.0000	260.74
D♭	Subdominant Minor Sixth	16/15	1.0667	278.13
D	Second (Supertonic)	9/8	1.1250	293.33
E♭	Minor Third	6/5	1.2000	312.89
E	Major Third	5/4	1.2500	325.93
F	Fourth (Subdominant)	4/3	1.3333	347.65
F♯	Dominant Major Seventh	45/32	1.4063	366.69
G	Fifth (Dominant)	3/2	1.5000	391.11
A♭	Minor Sixth	8/5	1.6000	417.18
A	Major Sixth	5/3	1.6667	434.57
B♭	Minor Seventh	9/5	1.8000	469.33
B	Major Seventh	15/8	1.8750	488.89
C	Octave (Tonic)	2/1	2.0000	521.48

In connection with research and experimental work on an electronic type piano, the writer devised the Electronichord as described in the *American Physics Teacher*, February, 1934. Using two Electronichords, it was possible to study the relations that exist between the harmonic overtones

of two string tones tuned any harmonic or tempered interval apart. These experiments, among other things, audibly demonstrated the importance of the harmonic intervals to satisfy the discriminating musical ear. The purpose of this paper is to offer a mathematical approach in demonstrating that the musical scale of just intonation is a fundamental acoustical measurement.

23. Incremental Coupled Musical Scales. M. H. GWYNN, *Technical Consultant, F.A.I.C., Mountain Lakes, New Jersey* (15 min.).—The equation $V/T=2^SN$ represents a family of preferred number series. S and N are of integer value, V is a rational number. One particular type series, which is both incremental and coupled, is defined by the following proportion for the terminal terms, i.e., extreme notes, of each scale, $N_{max}/N_{min}=2(S+C)/(S+C)$. C is an integer constant for each series of the type. Each of several incremental coupled series is an homologous train of scales whose notes are interpreted as frequencies, the ratios thereof being rational. Each scale possesses one more note than the preceding scale, and the highest frequency in each scale is an overlapping member of two arithmetic progressions. An octave is spanned within each scale. The lower scales of these series are separated in four groups. The several lowest scales compose two primitive and subsonic groups relating to tone quality. The next or sonic group consists of nine audible scales. Harmonic potentialities with respect to a new pure tone music are tabulated. One half of the sonic group notes are members of at least one major chord. Composition and rendition should be extraordinarily simple, since bass scales contain relatively few notes conforming to the recent findings that aural sensitivity to pitch progresses with sonic frequency.

24. Patents from the Viewpoint of the Applied Physicist. H. S. KNOWLES, *Jensen Radio Manufacturing Company, Chicago, Illinois* (20 min.).—Negligible attention is given to patents in most colleges because of the crowded curriculum and the widespread feeling that inventions should be treated in the same manner as scientific discoveries. As a consequence, many physicists and engineers are insufficiently instructed in the basic patent knowledge required in the academic and industrial fields and are not prepared to properly influence pending and future legislation on this subject. The journal of the Society has devoted an increasing amount of space to the publication of its review of acoustical patents. It therefore appears desirable to summarize some of the principal aspects of patents and their management from the viewpoint of the applied physicists. Stress is placed on sources of patent information, the selection of patent counsel, industrial patent practices and judicial and legislative trends.

25. Aero-Otitis Media in Submarine Personnel. HENRY HAINES, *Sound Laboratory, Medical Research Department, U. S. Submarine Base, New London, Connecticut* (15 min.).—Careful pre- and post-examination of several thousand men passing through the Submarine Escape Training Tank at New London reveals that about 30 percent have aero-otitis media as a result of exposure to the high pressure chamber. This agrees rather well with Fowler's figure of 22 percent among fliers in England. We believe the cause of the condition in most instances to be blockage of the Eustachian tube by hyperplastic adenoid tissue. A description of some of the effects of pressure on the ear is given from extensive data collected with the otoscope and with the nasopharyngoscope. A description is given of the effects of various types of treatment, principally radium applied to adenoids, x-ray, astringent nose drops, and psychological prescription.

26. Auditory Acuity in Severe Aero-Otitis Media. J. DONALD HARRIS, *Sound Laboratory, Medical Research Department, U. S. Submarine Base, New London, Connecticut.* (15 min.).—From personnel subjected to the Submarine Escape Training Tank at New London, a group was selected at random consisting of 90 men who were judged to have aero-otitis media of the severest, or No. 4, grade according to Dr. Teed's categories of otopathologic change. A brief description of the No. 4 category is given. Post-pressure audiograms at octaves 256 through 8,192 were compared with pre-pressure audiograms taken under almost identical conditions. Although a few cases exhibit a disabling loss, the mean drop for the whole group is surprisingly slight. It should be borne in mind that none of the cases reported here had been subjected to pressure more than once. Acuity seems not to drop markedly with perforation of the drumskin, bleeding from the drumskin or external meatus, or separation of the layers of the drumskin. It seems to be associated less with otopathology as such than with the extent to which the bulla is filled with blood. Thus the sharp drops in acuity which do occur may perhaps, at least in their early stages, be referred in large part to damping of the ossicles.

27. Anti-Noise Characteristics of Differential Microphones. H. E. ELLITHORN AND A. M. WIGGINS, *Electro-Voice Corporation, South Bend, Indiana* (15 min.).

28. Musicology, the Stepchild of the Sciences. ABE PEPINSKY, *Haverford College, Haverford, Pennsylvania* (15 min.).—The musicologist wants the respect of both the musician and the scientist, but the musician frowns upon him because, although he is supposed to know all about music, he seems to derive no sensory pleasure from it, and the scientist looks askance at the sort of stuff the musicologist busies himself with and his methods of approach. The question might well be asked: "What is Musicology?" Briefly, it purports to be the scientific study of music. Musicology reaches into many fields and someone facetiously remarked that it includes practically everything that lies outside of the personality of the performer and the purely technical aspects of his performance. But why exclude the personality of the performer or even the consideration of the technical aspects of the performance? Surely the personality of the performer plays an important role in presenting the musical thought to an auditor.

Likewise, the technical aspects of performance lend conviction to the meaning of the music. It would seem therefore, in considering music, that every possible phase of the art should be subjected to scrutiny and, if this be accomplished by utilizing acceptable scientific procedures, the musicologist need not be apologetic for his field of endeavor. Some of the areas explored will be reviewed together with the tools at the musicologist's command. New fields to conquer and the advantages of novel treatment will also be suggested.

29. Sound Conduction in the Head and in the Adjacent Regions. AGUSTA JELLINEK, *Polyclinic Hospital, New York, New York* (15 min.).—Sound conduction through bone and tissues varies according to the point of sound transmission. Every region of the head as well as neck and upper part of thorax has a conduction optimum for a certain pitch range, while other tones require greater intensities to be transmitted. Not only bony structures but also other tissues transmit sound at comparable intensities (for instance, throat and cheeks). The mastoids are not always the optimal regions of transmission, some people show better transmission from the temporal or occipital bones. Bone conduction varies far more with time than air conduction. For practical purposes these findings have some bearing on the use of hearing aids. Testing the bone conduction of the mastoids only gives an incomplete picture of the patient's possibilities. He may be able to hear better from another point. In some cases the hearing aid might be worn invisibly (on the skull, throat, occipital prominence). The acoustic selectivity of the head should be used in fitting a hearing aid.

30. Clinical Phenomena in Conductive Media: The Individual Earpiece. MAYER B. A. SCHIER, *New York, New York* (15 min.).—A hearing aid may be well designed and produced, acoustically and mechanically, but its optimum efficiency will depend upon delivery of instrument characteristics to the drum or middle ear structures by an earpiece. The initiated practice of fitting every air conduction hearing aid with an individual earpiece is an acceptable and generalized practice. This paper is concerned with clinical and experimental experiences on the effects produced by auditory prosthesis or the individually made earpiece as acoustic couplings and conductive medias; with divisional considerations of the many factors involved which may favorably or adversely induce maximum correlation between instrument and deafened ear.